Ideas and Non-Balanced Economic Growth

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Preliminary

Abstract

Employment, consumption expenditure, and R&D investment have grown faster in the service sector than in the manufacturing sector, resulting in the service-oriented economy. This paper shows that the difference in the idea production across sectors can cause the non-balanced sectoral growth. We build a two-sector growth model with endogenous technology. In this model, new ideas are produced using the current stock of ideas as an input, but the idea stock effect in idea production varies between sectors. Under the complementarity assumption between sectors, we show that this difference in idea production generates the non-balanced growth in R&D investment as well as in employment and consumption expenditure.

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In the post-industrial era, service jobs were supposed to replace the manufacturing jobs lost to foreign competition, much as manufacturing employment once replaced the farm jobs lost to higher agricultural productivity. To theorists of the 1950's, this meant a transition from an economy based on dirty heavy industry to one based on clean, well-lit offices. No more Pittsburghs... In the decade ahead, however, demand for services will slow down... productivity in the service sector will grow more rapidly while many fewer service jobs will be created...

— Lester C. Thurow (1989, Sep 04)

1. Introduction

Thurow's forecast about the service sector at the end of 1980s proved wrong. He saw the fast growth in service employment and demand as a temporal phenomenon. However, consumption expenditure in the service goods has not slowed down, and productivity growth in the service sector has not been fast enough to decrease the labor demand. The service sector's consumption expenditure and employment shares have steadily grown. The major economic activity that shifted from agriculture to manufacturing, has now shifted to service. Figure 1 shows this non-balanced growth across the sectors. In the growth literature, this is referred to as the *structural change* or *"Kuznets facts"* (Kuznets (1955, 1973), Kongsamut, Sergio and Xie (2001)).

Furthermore, this shift to the service sector extends beyond consumption expenditure and employment. For example, we can also find the same trend in the Research and Development(R&D) related statistics such as the number of scientists and R&D expenditures (Figure 2). This R&D trend seems to contradict the sectoral consumption and employment trends. One might expect that rapid R&D expenditure growth in services would push up productivity *growth* much more in the service sector than in the other sectors, as Thurow expected. Faster service productivity growth should in turn lead to slower service employment growth. However, there has been neither the slower service employment growth (Figure 1) nor the faster service productivity growth (Wölfl (2003), Triplett and Bosworth (2004)).

Therefore, we can characterize the post-industrial era as the service-oriented econ-

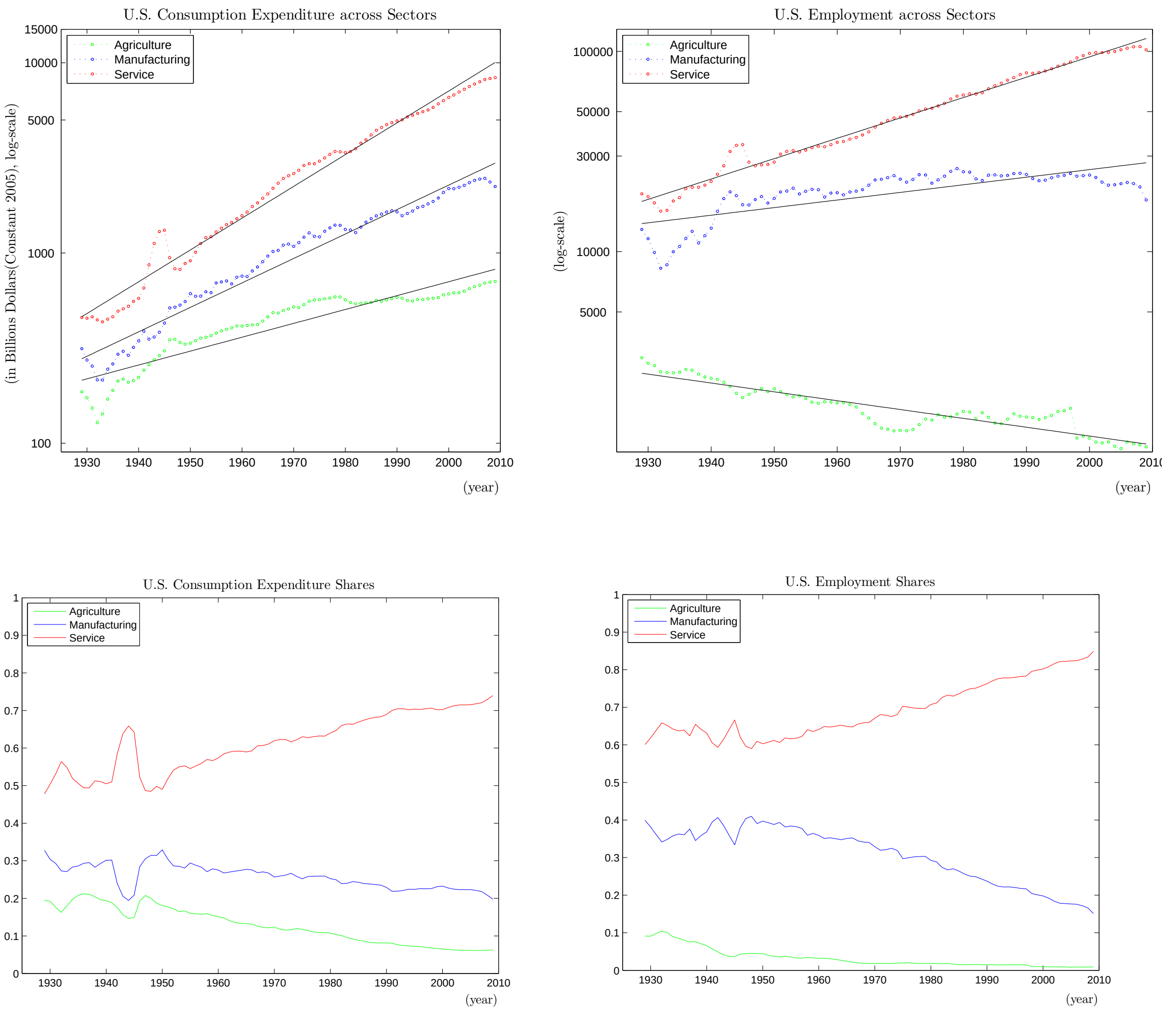

Figure 1: U.S. Sectoral Consumption Expenditure and Employment

Source: NIPA. See Appendix A for details.

omy where fundamental economic resources flow into the service sector. Then, why is it services, not manufacturing or agriculture? What is the main driving force behind this economic shift to the service sector? Can we also solve the puzzle of the R&D trend?

This paper argues that the key to answering these questions is the difference in the idea production across sectors. Similar to the role it plays in generating growth in idea-based growth models,[1] idea production plays a central role in generating structural change in our model. In particular, we pay attention to the fact that the structural change has been not only in consumption expenditure and employment, but also in R&D expenditure, which is closely related to new idea production.

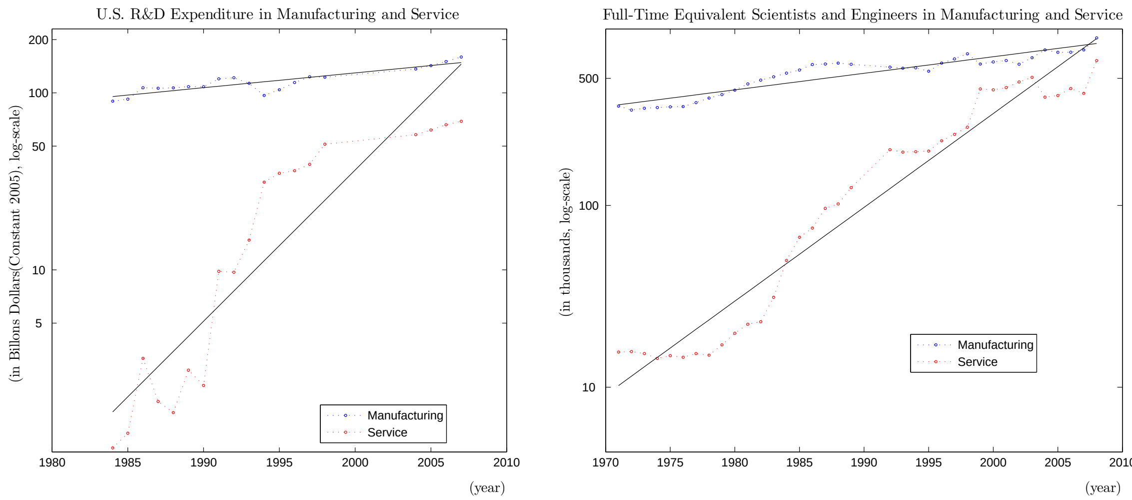

Figure 2: U.S. Sectoral R&D Expenditure and the R&D Employment

Source: NSF. See Appendix A for details.

We start from the assumption that more R&D will lead to more idea production. Secondly, new ideas are produced from the existing idea stock as in the idea-based growth models. The new idea production can be either proportional or inversely proportional to the size of the existing idea stock. When it is proportional, the higher level of idea stock leads to more new ideas, which is known as the *"standing on shoulders effect"*. When it is inversely proportional, the larger idea stock generates fewer new ideas, which is known as the *"fishing out effect"*. Throughout this paper, we refer to these two effects of the existing idea stock on the new idea production together as the *"idea stock*

[1] Jones (2005) reviews these models.

effect".

We further assume that the degree of this idea stock effect is different across sectors. This assumption is based on observing differences in the knowledge spillover effect across sectors. For example, if the technologies are more closely related in manufacturing than in services, it is clear that the impact of an idea from one manufacturing industry on other manufacturing industries is bigger than the impact of an idea from one service industry on other service industries. In other words, the manufacturing sector has a greater knowledge spillover effect within the sector than the service sector. This difference in the knowledge spillover effect in turn translates into the difference in the idea stock effect.

The greater idea stock effect in manufacturing appears more relevant than the greater idea stock effect in services since we would expect technologies in manufacturing are more homogeneous than technologies in services. Nevertheless, since the size of the idea stock effect has not yet been carefully estimated,[2] we examine both cases: the greater idea stock effect in manufacturing and the greater idea stock effect in services. We find that our model matches data only in case of the greater idea stock effect in manufacturing, which conforms to the intuition.

Incorporating this idea stock effect, we present a model where each sector's idea stock is augmented by R&D expenditures and the existing idea stock, and sectors by nature differ in the effect of the existing idea stock on the idea production. The model successfully generates the following stylized facts of structural change:**1**) **Employment, 2**) **Consumption Expenditure, and 3**) **R&D expenditure all grow faster in the service sector than in the manufacturing sector.**

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 develops an idea-based growth model with two sectors with different idea production functions, and then presents the non-balanced limiting behaviors of the optimal economy and of competitive equilibrium. Section 4 discusses implications of the model. Section 5 provides a simple calibration to verify that the model prediction is consistent with existing work on the labor productivity growth. Finally, Section 6 concludes.

[2]Ngai and Samaniego (2009) estimate the knowledge spillover effect for the selected industries, which are mostly classified as manufacturing.

2. Literature Review

In the literature, there are two main approaches to explain structural change, the demand-side approach and the supply-side approach. The demand-side approach explains the change by introducing a special type of consumer preference, while the supply-side approach generates the structural change by allowing different rates of technological progress across sectors.

The Demand Side Approach

The first attempt to explain the structural change from the consumer preference traces back to Engel (1857). The well-known Engel's law states that food consumption declines as a household income increases. This difference in the income elasticity of demand across goods is captured in the non-homothetic utility function. The approach of Kongsamut, Sergio and Xie (2001) is one example of introducing the non-homothetic preference (Stone-Geary preference) to generate structural change. However, the non-balanced growth in Kongsamut, Sergio and Xie (2001) depends on the knife-edge condition. In addition, there are "hierarchy of needs" models (Matsuyama (2002), **?**, Foellmi and Zweimuller (2008)), the consumer utility of which reflects that people need to consume food before textiles, textiles before electronics, and electronics before luxurious services.

The Supply Side Approach

The supply side approach to explain non-balanced growth across sectors dates back to Baumol (1967), who noted the importance of different productivity growth across sectors. This difference leads to changes in relative prices, then the relative price effect becomes the dominant force behind changes in consumption shares. This Baumol's idea appears in Ngai and Pissarides (2007), who provide a multi-sector growth model with different sectoral productivity growth rates. Acemoglu and Guerrieri (2006) generate non-balanced growth in a two-sector model from the assumption of different factor shares in the production function.

The recent empirical work by Herrendorf, Rogerson and Valentinyi (2009) finds that either the income effect or the relative effect can serve as the dominant force behind changes in expenditure shares depending on which expenditure specification one use in the model. There are also some recent works suggesting a unified model or introducing new forces behind structural change. **?** suggest that models with home production, sector-specific factor distortions, and different human capital accumulation are promising. Buera and Kaboski (2008) propose a unified model with sequential non-homotheticities and different scale technologies. **?** explain the rise of the service sector with a model which features sequential non-homotheticities, home production and the labor division to the high- and low-skilled labor. Desmet and Rossi-Hansberg (2010) introduce transport costs and spatial technology diffusion to generate the structural dynamics in manufacturing and service.

This paper takes the supply side approach to focus on the role of idea production in structural change. Moreover, our model can be seen as an endogenous version of Ngai and Pissarides (2007), who assumed an exogenous difference in technological progress across sectors. Endogenous technological progress enables us to explain why we have different technological progress across sectors. The main contribution of this paper is that it goes further to explain the richer dynamics of structural change, such as the different R&D expenditure growth across sectors, which has not captured in the literature.

3. The Two Sector Model

We present a Romer-style idea-based growth model but with the two sectors - manufacturing and service - where each sector has a different idea production function. The economy of this model consists of a representative household, a final good producer, an intermediate good producer in each sector, and an R&D sector.

The representative household provides labor and consumes final goods. The final good producer produces the final goods using the intermediate goods from the two sectors, and an intermediate good producer in each sector produces intermediate goods using machines purchased from the R&D sector. We assume both the final goods market and the intermediate goods market are perfectly competitive. The R&D sector produces ideas for the new machine varieties both for the intermediate manufacturing goods and the intermediate service goods.

The idea stock in each sector is augmented by R&D expenditures[3] and the existing level of idea stock in the sector. The exclusive right to produce and sell the particular machine is given to the R&D sector, and thereby the monopoly profits create the incentive to discover an idea for a new machine variety. But this incentive, or the value of a new idea can differ between the two sectors due to the difference in the idea production function. This becomes the driving force for non-balanced growth in this economy.

We now begin by laying out the economic environment in detail, and then we will solve for the optimal allocation and the competitive equilibrium.

3.1. The Economic Environment

Throughout this paper let the subscript $i \in \{m, s\}$ denote the two sectors in the economy, where m and s represent the manufacturing sector and the service sector respectively.

The representative household provides labor $L(t)$, which grows with the rate n, and consumes $C(t)$. The representative household has a standard CRRA preference over the infinite time horizon:

$$U_t = \int_{\tau=t}^{\infty} e^{-\rho\tau} \frac{C(\tau)^{1-\sigma} - 1}{1-\sigma} d\tau, \tag{1}$$

where $1/\sigma$ is the elasticity of intertemporal substitution.

The production functions in this economy are

$$Y(t) = \left[\gamma Y_m(t)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) Y_s(t)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

$$Y_i(t) = \left(\int_0^{A_i(t)} z_i(\nu, t)^{\theta} d\nu\right)^{\frac{1}{\theta}}, \forall i \in \{m, s\}, \tag{3}$$

$$\int_0^{A_i(t)} z_i(\nu, t) d\nu = L_i(t), \forall i \in \{m, s\}, \tag{4}$$

$$\dot{A}_i(t) = \eta_i A_i(t)^{\phi_i} X_i(t), \forall i \in \{m, s\}. \tag{5}$$

(2) is the production function for the final good $Y(t)$. $Y(t)$ is produced using the intermediate goods $Y_m(t)$ and $Y_s(t)$. Here we assume that the manufacturing goods

[3] This is a lab-equipment setting. The model where more scientists create more ideas is presented in Appendix B, and the results are very similar.

and service goods are complementary, so that the elasticity of substitution ϵ is less than 1.

In (3), the intermediate good $Y_i(t)$ is produced using the collection of machines $z_i(\nu, t)$ of variety ν in the sector i. $A_i(t)$ represents the measure of available machine varieties or the level of idea accumulation in the sector i. Note that $A_i(t)$ can also translate into the productivity of the sector i. The machine varieties $z_i(\nu, t)$ enter the intermediate good production function through a CES aggregator. We assume the elasticity of substitution among machine varieties $1/(1-\theta)$ is greater than 1 $(0 < \theta < 1)$ because it is natural to think that new machines can substitute for the existing machines.

Regarding the machine production, we assume that labor $L_i(t)$ is the only input. This assumption may contradict a common sense that a machine is one form of physical capital, but note that one machine variety in our model is a realization of an idea rather than a form of capital. We further assume that one unit of labor produces one unit of machines. Due to this one-to-one transformative characteristic of the machine production, (4) serves as a machine production function as well as a labor resource constraint.

Next, each sector has the idea production function (5). The number of new ideas in sector i for designs of new machine varieties is increasing in the R&D expenditure $X_i(t)$, and is also related to the existing stock of knowledge $A_i(t)$. ϕ_i allows the idea production in the R&D sector to be either an increasing $(\phi_i > 0)$ or a decreasing $(\phi_i < 0)$ function of the existing idea stock. In other words, ϕ_i captures the *idea stock effect*, the effect of the existing idea stock on the new idea production - the *standing on shoulders effect* $(\phi_i > 0)$ or the *fishing out effect* $(\phi_i < 0)$. We let the two sectors differ in ϕ_i, which takes into account that the idea production processes can be different due to the differences in the technologies of each sector. For example, we see that the technologies in the manufacturing sector are more related to each other than the technologies in the service sector because the service sector features the higher level of heterogeneity within the sector. Consequently, it is more likely that researchers in the manufacturing sector will get some help from the existing idea stock in the same sector when they invent new technologies than researchers in the service sector. That is, the manufacturing sector will have a greater standing on shoulders effect or smaller fishing out effect $(\phi_m > \phi_s)$. We will later see that this is the fundamental driving force behind the non-balanced growth. We further assume $\phi_i < 1$, which is consistent with Acemoglu and Guerrieri (2008).

Lastly, the resource constraints for this economy are

$$Y(t) = C(t) + X_m(t) + X_s(t), \tag{6}$$

$$L_m(t) + L_s(t) = L(t) = L_0 e^{nt}, \tag{7}$$

$$L_i(t) = \int_0^{A_i(t)} z_i(\nu, t) d\nu, \forall i \in \{m, s\}, \tag{8}$$

where $C(t)$ is the consumption of the representative household and n is the growth rate of population. (6) states that the final goods can be either consumed or used as R&D expenditure. Note that although capital is absent in the model, R&D expenditure can be interpreted as investment for the future consumption. (7) is the usual labor resource constraint. Lastly, (8) is the sector i's labor resource constraint, which is identical to (4).

3.2. The Optimal Allocation

Given this economic environment, we first study the allocation of the economy that maximizes welfare of the representative household, which is defined as follows:

Definition The *optimal allocation* in this economy consists of time paths for $\{Y(t)$, $C(t)\}_{t=0}^{\infty}$, $\{Y_i(t)$, $A_i(t)$, $L_i(t)$, $X_i(t)$, $\{z_i(\nu, t)\}_{\nu \in [0, A_i(t)]}\}_{t=0, i \in \{m,s\}}^{\infty}$ that maximize utility U_t in (1) given the economic environment (2)-(7).

To solve the optimal allocation, we adopt the perspective of an imaginary social planner whose objective is to maximize the total welfare of the economy. We have 12 unknown allocation objects in the social planner's problem of this economy, but we can reduce the problem by replacing some variables with the equality conditions in (2)-(7).

First, using the symmetry among machine varieties, we have $z_i(\nu, t) = z_i(t)$. Then

(3) and (4) imply

$$L_i(t) = \int_0^{A_i(t)} z_i(\nu,t)d\nu = A_i(t)z_i(t), \quad (9)$$

$$Y_i(t) = \left(\int_0^{A_i(t)} z_i(t)^\theta d\nu\right)^{\frac{1}{\theta}} = \left(A_i(t)z_i(t)^\theta\right)^{\frac{1}{\theta}} = A_i(t)^{\frac{1-\theta}{\theta}}L_i(t). \quad (10)$$

Thus, if $A_i(t)$ and $L_i(t)$ are predetermined, $z_i(t)$ and $Y_i(t)$ will follow by the above equations. This $Y_i(t)$ in turn determines $Y(t)$ by (2). $Y(t)$ together with $X_m(t)$ and $X_s(t)$ gives the allocation of $C(t)$ by (6). Also, $L_s(t)$ is fixed by (7) if $L_m(t)$ is known. In sum, once the social planner optimally solves for $\{A_i(t), X_i(t)\}_{i\in\{m,s\}}$ and $L_m(t)$, the other allocation variables are automatically determined by (2), (3), (4), (6) and (7). Note that we have not used (5) so that this has to be included in the reduced problem. We now define the reduced social planner's problem as follows.

Social Planner's Problem(SP'): the social planner sets the optimal time paths of allocation

$$\{A_i(t), X_i(t)\}_{t=0,i\in\{m,s\}}^\infty, \{L_m(t)\}_{t=0}^\infty$$

which solve

$$\max_{\{X_m(t),X_s(t),L_m(t)\}_{t=0}^\infty} \int_0^\infty e^{-\rho t}\frac{C(t)^{1-\sigma}-1}{1-\sigma}dt$$

$$\begin{aligned}
\text{where} \quad C(t) &= \left[\gamma\left(A_m(t)^{\frac{1-\theta}{\theta}}L_m(t)\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)\left(A_s(t)^{\frac{1-\theta}{\theta}}(L(t)-L_m(t))\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \\
&\quad -X_m(t) - X_s(t) \\
\text{s.t.} \quad \dot{A}_i(t) &= \eta_i A_i(t)^{\phi_i}X_i(t), \forall i \in \{m,s\}
\end{aligned}$$

Now the problem has 5 unknowns(3 control variables $L_m(t)$, $X_m(t)$, $X_s(t)$, and 2 state variables $A_m(t)$, $A_s(t)$) and 5 equations to determine these unknowns at each point in time. To solve SP', we begin by setting up the current-value Hamiltonian,

$$\mathcal{H} = \frac{1}{1-\sigma}\left\{([\gamma(A_m(t)^{\frac{1-\theta}{\theta}}L_m(t))^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)(A_s(t)^{\frac{1-\theta}{\theta}}(L(t)-L_s(t)))^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} - X_m(t) - X_s(t))^{1-\sigma} - 1\right\} + \lambda_m(t)\eta_m A_m(t)^{\phi_m}X_m(t) + \lambda_s(t)\eta_s A_s(t)^{\phi_s}X_s(t)$$

Then the first-order conditions are given by

$$\frac{\partial\mathcal{H}}{\partial X_m(t)} = 0 \quad : \quad C(t)^{-\sigma} = \lambda_m(t)\eta_m A_m(t)^{\phi_m}, \tag{11}$$

$$\frac{\partial\mathcal{H}}{\partial X_s(t)} = 0 \quad : \quad C(t)^{-\sigma} = \lambda_s(t)\eta_s A_s(t)^{\phi_s}, \tag{12}$$

$$\frac{\partial\mathcal{H}}{\partial L_m(t)} = 0 \quad : \quad \frac{L_s(t)}{L_m(t)} = \frac{(1-\gamma)^\epsilon}{\gamma^\epsilon}\left(\frac{A_m(t)}{A_s(t)}\right)^{\frac{1-\theta}{\theta}(1-\epsilon)}, \tag{13}$$

$$\frac{\partial\mathcal{H}}{\partial A_m(t)} = \rho\lambda_m(t) - \dot{\lambda}_m(t) \quad : \quad \gamma\frac{1-\theta}{\theta}\eta_m A_m(t)^{\phi_m-1}\left(\frac{Y_m(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}}Y_m(t) = \rho - \frac{\dot{\lambda}_m(t)}{\lambda_m(t)} - \eta_m\phi_m A_m(t)^{\phi_m-1}X_m(t), \tag{14}$$

$$\frac{\partial\mathcal{H}}{\partial A_s(t)} = \rho\lambda_s(t) - \dot{\lambda}_s(t) \quad : \quad (1-\gamma)\frac{1-\theta}{\theta}\eta_m A_s(t)^{\phi_s-1}\left(\frac{Y_s(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}}Y_s(t) = \rho - \frac{\dot{\lambda}_s(t)}{\lambda_s(t)} - \eta_s\phi_s A_s(t)^{\phi_s-1}X_s(t). \tag{15}$$

In addition, we have the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_m(t)A_m(t) = 0, \tag{16}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_s(t)A_s(t) = 0. \tag{17}$$

Let us next define a *constant growth path*(CGP) as a trajectory of the economy where the growth rate of each variable converges to a constant, but possibly with different values. In other words, if we define $g_{ij}(t) \equiv \frac{\dot{I}_j(t)}{I_j(t)}$, $g_{ij}^* \equiv \lim_{t\to\infty} g_{ij}(t)$, the finite value of g_{ij}^* for each variable exists along a CGP. We now solve for the optimal allocation along a CGP and corresponding limiting growth rates.

1. Optimal allocation of labor and the limiting growth rates

(7) and (13) give the optimal allocation of labor:

$$
\begin{aligned}
L_m(t) &= \frac{\gamma^\epsilon}{(1-\gamma)^\epsilon \left(\frac{A_s(t)}{A_m(t)}\right)^{-\frac{1-\theta}{\theta}(1-\epsilon)} + \gamma^\epsilon} L(t), \\
L_s(t) &= \frac{(1-\gamma)^\epsilon}{(1-\gamma)^\epsilon + \gamma^\epsilon \left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}(1-\epsilon)}} L(t).
\end{aligned} \tag{18}
$$

Labor shares follow directly from (18). Thus, $\frac{L_m(t)}{L(t)} = \gamma^\epsilon/((1-\gamma)^\epsilon(A_s(t)/A_m(t))^{-\frac{1-\theta}{\theta}(1-\epsilon)} + \gamma^\epsilon)$ and $\frac{L_s(t)}{L(t)} = (1-\gamma)^\epsilon/((1-\gamma)^\epsilon + \gamma^\epsilon(A_s(t)/A_m(t))^{\frac{1-\theta}{\theta}(1-\epsilon)})$. Also, the limiting growth rates of the labor allocations are obtained from log-differencing (18):

$$
\begin{aligned}
&\text{if } g^*_{am} > g^*_{as}, \quad g^*_{lm} = n - \tfrac{1-\theta}{\theta}(1-\epsilon)(g^*_{am} - g^*_{as}) < g^*_{ls} = n, \\
&\text{if } g^*_{as} > g^*_{am}, \quad g^*_{lm} = n < g^*_{ls} = n - \tfrac{1-\theta}{\theta}(1-\epsilon)(g^*_{as} - g^*_{am}).
\end{aligned} \tag{19}
$$

Next, the labor shares converge as follows:

$$
\begin{aligned}
&\text{if } g^*_{am} > g^*_{as}, \quad \lim_{t\to\infty} \tfrac{L_m(t)}{L(t)} = 1, \lim_{t\to\infty} \tfrac{L_s(t)}{L(t)} = 0, \\
&\text{if } g^*_{as} > g^*_{am}, \quad \lim_{t\to\infty} \tfrac{L_m(t)}{L(t)} = 0, \lim_{t\to\infty} \tfrac{L_s(t)}{L(t)} = 1.
\end{aligned} \tag{20}
$$

We see that the time path of the productivity ratio $\frac{A_s(t)}{A_m(t)}$ governs the time paths of labor allocations and hence the labor share. $\frac{A_s(t)}{A_m(t)}$ goes to 0 if the productivity of service $A_s(t)$ grows at a slower rate than the productivity of manufacturing $A_m(t)$, and goes to ∞ if the opposite is the case. Therefore, if $g^*_{as} < g^*_{am}$, then $\frac{A_s(t)}{A_m(t)}$ converges to 0, which in turn implies that $L_s(t)$ grows as fast as the total labor $L(t)$ and $L_m(t)$ grows at a slower rate than the total labor $L(t)$. Subsequently, the labor share of manufacturing goes to 0 and that of service goes to 1. The opposite occurs if $g^*_{as} > g^*_{am}$. In other words, if productivity grows at different rates between sectors, it is optimal for the production input, here labor, to be reallocated to the slow growing sector, and hence the labor share of the fast growing sector goes to 0.

2. Optimal allocation of R&D expenditures and the limiting growth rates

From (5), we know the limiting growth rate of productivity is given by

$$g_{ai} = \frac{\dot{A}_i(t)}{A_i(t)} = \eta_i A_i(t)^{\phi_i - 1} X_i(t).$$

Along a CGP, the right-hand side of this equation should converge to a constant, so we obtain the relationship between the limiting growth rates of R&D expenditure and productivity,

$$g^*_{xm} = (1-\phi_m) g^*_{am}, \quad g^*_{xs} = (1-\phi_s) g^*_{as}. \tag{21}$$

Also from (11) and (12),

$$\frac{\dot{\lambda}_i(t)}{\lambda_i(t)} = \sigma g_c + \phi_i g_{ai}$$

Using this result, (5), (14) and (15), the R&D expenditures are given by

$$\begin{aligned} X_m(t) &= \frac{\gamma \frac{1-\theta}{\theta} g_{am} \left(\frac{Y_m(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}} Y_m(t)}{\rho + \sigma g_c}, \\ X_s(t) &= \frac{(1-\gamma) \frac{1-\theta}{\theta} g_{as} \left(\frac{Y_s(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}} Y_s(t)}{\rho + \sigma g_c} \end{aligned} \tag{22}$$

This together with (13) gives the ratio of the R&D expenditure in two sectors,

$$\frac{X_m(t)}{X_s(t)} = \frac{\gamma^\epsilon}{(1-\gamma)^\epsilon} \frac{g_{am}}{g_{as}} \left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}(1-\epsilon)} = \frac{g_{am}}{g_{as}} \frac{L_m(t)}{L_s(t)}, \tag{23}$$

The R&D expenditure ratio $\frac{X_m(t)}{X_s(t)}$ is inversely related to the productivity ratio $\frac{A_s(t)}{A_m(t)}$, as is the labor ratio $\frac{L_m(t)}{L_s(t)}$. That is, as the productivity gap widens, the labor and R&D expenditure gap also becomes larger but in the opposite direction. If $\frac{A_s(t)}{A_m(t)}$ converges to 0 or ∞ because the productivity grows at different rates, (23) further implies that $X_s(t) \gg X_m(t)$ if $g^*_{am} > g^*_{as}$, and $X_m(t) \gg X_s(t)$ if $g^*_{am} < g^*_{as}$. Thus, we can conclude that it is optimal to invest more in the less advanced sector. But also note that this is not enough to boost the productivity growth of the slow sector to at least the same level as the other sector. We will later discuss the R&D expenditure shares in detail.

Now, we differentiate (49) with respect to time and use (21) to get the following relationship among the limiting growth rates:

$$
\begin{aligned}
& g_{xs}^* - g_{xm}^* = (1-\phi_s)g_{as}^* - (1-\phi_m)g_{am}^* = g_{ls}^* - g_{lm}^* = \frac{1-\theta}{\theta}(1-\epsilon)(g_{am}^* - g_{as}^*) \\
\Rightarrow \quad & \frac{g_{am}^*}{g_{as}^*} = \frac{1-\phi_s + \frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m + \frac{1-\theta}{\theta}(1-\epsilon)} \\
\Rightarrow \quad & g_{am}^* > g_{as}^* \quad \text{if and only if } 1 > \phi_m > \phi_s, \\
& g_{am}^* < g_{as}^* \quad \text{if and only if } \phi_m < \phi_s < 1.
\end{aligned}
$$

This result shows that when $\epsilon < 1$, productivity grows faster in the sector with high ϕ than the other sector.

Again from (14), we have

$$
\begin{aligned}
\gamma \tfrac{1-\theta}{\theta}\eta_m A_m(t)^{\phi_m-1}\left(\frac{Y_m(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}} Y_m(t) = \;&= \rho - \tfrac{\dot{\lambda}_m(t)}{\lambda_m(t)} - \eta_m\phi_m A_m(t)^{\phi_m-1}X_m(t) \\
\to \gamma\eta_m \tfrac{1-\theta}{\theta} Y(t)^{\frac{1}{\epsilon}} L_m(t)^{\frac{\epsilon-1}{\epsilon}} A_m(t)^{\phi_m-1+\frac{1-\theta}{\theta}(1-\frac{1}{\epsilon})} \;&= \rho - \tfrac{\dot{\lambda}_m(t)}{\lambda_m(t)} - \phi_m g_{am}.
\end{aligned}
$$

Similarly (15) becomes

$$
(1-\gamma)\eta_s \tfrac{1-\theta}{\theta} Y(t)^{\frac{1}{\epsilon}} L_s(t)^{\frac{\epsilon-1}{\epsilon}} A_s(t)^{\phi_s-1+\frac{1-\theta}{\theta}(1-\frac{1}{\epsilon})} = \rho - \tfrac{\dot{\lambda}_s(t)}{\lambda_s(t)} - \phi_s g_{as}.
$$

The limiting growth rate of the left-hand side of the above equations should be zero since the right-hand side converges to a constant. Therefore we obtain

$$
\begin{aligned}
\tfrac{1}{\epsilon}g_y^* + \tfrac{\epsilon-1}{\epsilon}g_{lm}^* + (\phi_m - 1 + \tfrac{1-\theta}{\theta}\left(1-\tfrac{1}{\epsilon}\right))g_{am}^* = 0, \\
\tfrac{1}{\epsilon}g_y^* + \tfrac{\epsilon-1}{\epsilon}g_{ls}^* + (\phi_s - 1 + \tfrac{1-\theta}{\theta}\left(1-\tfrac{1}{\epsilon}\right))g_{as}^* = 0.
\end{aligned}
\tag{24}
$$

Also note that when $\epsilon < 1$, from (2),

$$
\begin{aligned}
g_y^* &= \min\{g_{ym}^*, g_{ys}^*\} \\
&= \min\left\{\tfrac{1-\theta}{\theta} g_{am}^* + g_{lm}^*, \tfrac{1-\theta}{\theta} g_{as}^* + g_{ls}^*\right\} \\
&= \begin{cases} \min\{\frac{1-\theta}{\theta}(\epsilon g_{am}^* + (1-\epsilon) g_{as}^*) + n, \frac{1-\theta}{\theta} g_{as}^* + n\} = \frac{1-\theta}{\theta} g_{as}^* + n = g_{ys}^*, & \text{if } \phi_m > \phi_s \\ \min\{\frac{1-\theta}{\theta} g_{am}^* + n, \frac{1-\theta}{\theta}(\epsilon g_{as}^* + (1-\epsilon) g_{am}^*) + n\} = \frac{1-\theta}{\theta} g_{am}^* + n = g_{ym}^*, & \text{if } \phi_m < \phi_s. \end{cases}
\end{aligned} \tag{25}
$$

Combining this result with (21) and (24), we finally obtain the limiting growth rates of productivity and R&D expenditure,

$$
\text{if } \phi_m > \phi_s, \quad \begin{cases} g_{am}^* = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}} \frac{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)} n & > g_{as}^* = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}} n, \\ g_{xm}^* = \frac{1-\phi_m}{1-\phi_s-\frac{1-\theta}{\theta}} \frac{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)} n & < g_{xs}^* = \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}} n = g_y^*, \end{cases}
$$

$$
\text{if } \phi_s > \phi_m, \quad \begin{cases} g_{am}^* = \frac{1}{1-\phi_m-\frac{1-\theta}{\theta}} n & < g_{as}^* = \frac{1}{1-\phi_m-\frac{1-\theta}{\theta}} \frac{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)} n, \\ g_{xm}^* = \frac{1-\phi_m}{1-\phi_m-\frac{1-\theta}{\theta}} n = g_y^* & > g_{xs}^* = \frac{1-\phi_s}{1-\phi_m-\frac{1-\theta}{\theta}} \frac{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)} n. \end{cases}
$$

This result states that in a sector with high ϕ, productivity grows at a faster rate, while the R&D expenditure grows at a slower rate in the limit. This in turn implies that the R&D expenditure share of the sector will converge to 0. To see this in more detail, let $X(t)$ denote the total R&D expenditure $X_m(t) + X_s(t)$. Then from (22), the R&D expenditure shares are given by

$$
\begin{aligned}
\frac{X_m(t)}{X(t)} &= \frac{1}{1+\frac{1-\gamma}{\gamma}\frac{g_{as}}{g_{am}}\left(\frac{Y_m(t)}{Y_s(t)}\right)^{\frac{1-\epsilon}{\epsilon}}}, \\
\frac{X_s(t)}{X(t)} &= \frac{1}{1+\frac{\gamma}{1-\gamma}\frac{g_{am}}{g_{as}}\left(\frac{Y_m(t)}{Y_s(t)}\right)^{-\frac{1-\epsilon}{\epsilon}}}.
\end{aligned} \tag{26}
$$

From (25), we know that $\frac{Y_m(t)}{Y_s(t)}$ goes to 0 if $\phi_m > \phi_s$, and to ∞ if $\phi_m < \phi_s$. Thus, the R&D

expenditure shares converge as follows:

$$\begin{aligned} &\text{if } g_{am}^* > g_{as}^* (\Leftrightarrow \phi_m > \phi_s), \quad \lim_{t\to\infty} \frac{X_m(t)}{X(t)} = 1, \lim_{t\to\infty} \frac{X_s(t)}{X(t)} = 0, \\ &\text{if } g_{am}^* < g_{as}^* (\Leftrightarrow \phi_m < \phi_s), \quad \lim_{t\to\infty} \frac{X_m(t)}{X(t)} = 0, \lim_{t\to\infty} \frac{X_s(t)}{X(t)} = 1. \end{aligned} \tag{27}$$

Next, we get the limiting growth rates of labor in terms of the exogenous parameters by substituting g_{am}^* and g_{as}^* back into (19)[4], If $\phi_m > \phi_s$,

$$\begin{aligned} g_{lm}^* &= \left(1 - \frac{1-\theta}{\theta} \frac{\phi_m - \phi_s}{1 - \phi_m + \frac{1-\theta}{\theta}(1-\epsilon)} \frac{1}{1 - \phi_s - \frac{1-\theta}{\theta}}\right) n \\ g_{ls}^* &= n. \end{aligned}$$

Lastly, notice that in the equations for the limiting growth rates we need to impose the restriction $1 - \frac{1-\theta}{\theta} - \phi_{min} > 0$ to have a positive growth path, where $\min\{\phi_m, \phi_s\} = \phi_{min}$ and $\max\{\phi_m, \phi_s\} = \phi_{max}$.

3. Optimal Consumption and its limiting growth rate

The optimal consumption can be attained from (6),

$$\begin{aligned} C(t) &= Y(t) - X_m(t) - X_s(t) \\ &= \left[\gamma \left(A_m(t)^{\frac{1-\theta}{\theta}} L_m(t)\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) \left(A_s(t)^{\frac{1-\theta}{\theta}} L_s(t)\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \\ &\quad - \frac{\eta_m}{g_{am}} A_m(t)^{1-\phi_m} - \frac{\eta_s}{g_{as}} A_s(t)^{1-\phi_s} \end{aligned}$$

If $\phi_m > \phi_s$, we know from (26) that $Y(t)$ and $X_s(t)$ grow at the same rate while $X_m(t)$ grows at the slower rate($g_{xm}^* < g_{xs}^* = g_y^*$). Accordingly, $g_c^* \leq g_y^* = g_{xs}^*$. $g_y^* = g_{xs}^*$ means $Y(t)$ has at least one term which grows at a rate of g_{xs}^*, and this term is indeed the fastest growing term in $Y(t)$. We see that this fastest growing term in $Y(t)$ is in $A_s(t)^{\frac{1-\theta}{\theta}} L_s(t)$. We can have $g_c^* < g_y^*$ only when this term and the fastest growing

[4]It is worth noting that $1 - \frac{1-\theta}{\theta} - \phi_{min} > 0$ guarantees the positive growth except the labor in the sector with ϕ_{max}. It is not necessary to have the positive labor growth in both sectors, but if the parameters further satisfy $1 - \frac{1-\theta}{\theta} = 2 - \frac{1}{\theta} > \max\{\phi_m, \phi_s\} \Leftrightarrow 1 - \frac{1-\theta}{\theta} \frac{\phi_m - \phi_s}{1-\phi_m + \frac{1-\theta}{\theta}(1-\epsilon)} \frac{1}{1-\phi_s - \frac{1-\theta}{\theta}} > 0$, we have the positive growth equilibrium where every variable grows with a positive rate.

term in $X_s(t) = \frac{\eta_s}{g_{as}} A_s(t)^{1-\phi_s}$ are the exactly same including the coefficients.[5] Since this is clearly not the case, the consumption growth rate g_c^* must be equal to the output growth rate g_y^*. The same argument can be made in the case of $\phi_s > \phi_m$. Thus, we obtain $g_c^* = g_y^*$.

4. Transversality conditions

To ensure transversality conditions (16) and (17) hold, we need to impose $-\rho + \frac{\dot{\lambda}_m(t)}{\lambda_m(t)} + g_{am}^* < 0$ and $-\rho + \frac{\dot{\lambda}_s(t)}{\lambda_s(t)} + g_{as}^* < 0$. These restrictions can be rewritten as:

$$
\begin{aligned}
& -\rho + \frac{\dot{\lambda}_m(t)}{\lambda_m(t)} + g_{am}^* < 0 \text{ and } -\rho + \frac{\dot{\lambda}_s(t)}{\lambda_s(t)} + g_{as}^* < 0 \\
& \Rightarrow (1-\phi_m) g_{am}^* - \sigma g_c^* < \rho \text{ and } (1-\phi_s) g_{as}^* - \sigma g_c^* < \rho \quad \left(\because \frac{\dot{\lambda}_i(t)}{\lambda_i(t)} = -\phi_i g_{ai}^* - \sigma g_c^* \text{ from (11),(12)}\right) \\
& \Rightarrow \max\{(1-\phi_m) g_{am}^*, (1-\phi_s) g_{as}^*\} - \sigma g_c^* < \rho \\
& \Rightarrow (1-\sigma) g_c^* < \rho \qquad \left(\because \max\{(1-\phi_m) g_{am}^*, (1-\phi_s) g_{as}^*\} = g_y^* = g_c^*\right) \\
& \Rightarrow (1-\sigma) \frac{1-\phi_{min}}{1-\phi_{min} - \frac{1-\theta}{\theta}} n < \rho
\end{aligned}
$$

5. The growth rate of the economy

Finally, it should be recognized that g_y^* and g_c^* are the growth rates of the *total* output and consumption, not the *per-capita* output and consumption. Since we normally evaluate the economy by how much one person can produce or consume, the growth rate of the economy should refer to the growth rate of output per person or consumption per person. This per-capita growth rate is easily obtained by subtracting the population growth rate from the growth rate of total output and consumption. Thus, if we let g^* denote the the growth rate of the economy, it is given by

$$
g^* = g_y^* - n = g_c^* - n = \frac{\frac{1-\theta}{\theta}}{1 - \phi_{min} - \frac{1-\theta}{\theta}} n.
$$

We can now summarize the limiting behavior of the optimal allocation along a CGP by the following theorem.

Theorem 1. Suppose $\epsilon < 1$, $n > 0$, $(1-\sigma)\frac{1-\phi_{min}}{1-\phi_{min}-\frac{1-\theta}{\theta}} n < \rho$ and $2 - \frac{1}{\theta} > \phi_{min}$. Then

[5] Note that $Y(t)$, $Y_s(t) = A_s(t)^{\frac{1-\theta}{\theta}} L_s(t)$, and $X_s(t) = \frac{\eta_s}{g_{as}} A_s(t)^{1-\phi_s}$ can also include other terms inside which grow with the slower rates.

there exists an optimal CGP such that

$$\text{if } \phi_m > \phi_s,\quad \begin{cases} g_{am}^* = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}\frac{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}n & > g_{as}^* = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}n \\ g_{xm}^* = \frac{1-\phi_m}{1-\phi_s-\frac{1-\theta}{\theta}}\frac{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}n & < g_{xs}^* = \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}}n \\ g_{lm}^* = \left(1-\frac{1-\theta}{\theta}\frac{\phi_m-\phi_s}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}\frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}\right)n & < g_{ls}^* = n \\ g_{ym}^* = \left((1-\phi_s)+\epsilon\frac{\phi_m-\phi_s}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}\right)\frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}n & > g_{ys}^* = \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}}n \\ g_y^* = g_{ys}^* = g_c^* = g_{xs}^* = \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}}n \\ g^* = \frac{\frac{1-\theta}{\theta}}{1-\phi_s-\frac{1-\theta}{\theta}}n \\ \lim_{t\to\infty}\frac{L_m(t)}{L(t)} = 0,\ \lim_{t\to\infty}\frac{L_s(t)}{L(t)} = 1 \\ \lim_{t\to\infty}\frac{X_m(t)}{X(t)} = 0,\ \lim_{t\to\infty}\frac{X_s(t)}{X(t)} = 1 \end{cases}$$

$$\text{if } \phi_s > \phi_m,\quad \begin{cases} g_{am}^* = \frac{1}{1-\phi_m-\frac{1-\theta}{\theta}}n & < g_{as}^* = \frac{1}{1-\phi_m-\frac{1-\theta}{\theta}}\frac{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}n \\ g_{xm}^* = \frac{1-\phi_m}{1-\phi_m-\frac{1-\theta}{\theta}}n & > g_{xs}^* = \frac{1-\phi_s}{1-\phi_m-\frac{1-\theta}{\theta}}\frac{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}n \\ g_{lm}^* = n & > g_{ls}^* = \left(1-\frac{1-\theta}{\theta}\frac{\phi_s-\phi_m}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}\frac{1}{1-\phi_m-\frac{1-\theta}{\theta}}\right)n \\ g_{ym}^* = \frac{1-\phi_m}{1-\phi_m-\frac{1-\theta}{\theta}}n & < g_{ys}^* = \left((1-\phi_m)+\epsilon\frac{\phi_s-\phi_m}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}\right)\frac{1}{1-\phi_m-\frac{1-\theta}{\theta}}n \\ g_y^* = g_{ym}^* = g_c^* = g_{xm}^* = \frac{1-\phi_m}{1-\phi_m-\frac{1-\theta}{\theta}}n \\ g^* = \frac{\frac{1-\theta}{\theta}}{1-\phi_m-\frac{1-\theta}{\theta}}n \\ \lim_{t\to\infty}\frac{L_m(t)}{L(t)} = 1,\ \lim_{t\to\infty}\frac{L_s(t)}{L(t)} = 0 \\ \lim_{t\to\infty}\frac{X_m(t)}{X(t)} = 1,\ \lim_{t\to\infty}\frac{X_s(t)}{X(t)} = 0 \end{cases}$$

Proof: The derivations of the limiting growth rates and the labor and R&D expenditure share convergence are given in the preceding discussion in this section. ■

To see the intuition of Theorem 1, suppose we have the greater *idea stock effect* in the manufacturing sector than in the service sector ($\phi_m > \phi_s$). This case appears as the more relevant benchmark, since we would expect that it is more likely that new ideas are created upon the existing ideas in manufacturing than in services at the macro level. New ideas in the service sector are created in a *relatively* independent way. For example, setting aside the knowledge spillover across sectors, the degree of influence of previous research in the same sector is larger on the invention of the computer than on the invention of Google's Internet search service.

Since we can easily build upon existing knowledge in the manufacturing sector ($\phi_m > \phi_s$), we have the faster idea accumulation rate in the manufacturing sector than the service sector ($g^*_{am} > g^*_{as}$). This higher productivity growth in manufacturing in turn leads to the faster production growth in the manufacturing sector ($g^*_{ym} > g^*_{ys}$). Here we should be careful *not* to think of $Y_m(t)$ and $Y_s(t)$ as the total *expenditure.*

Then this gap in productivity growth causes the share of labor in manufacturing to fall to balance out the two sectors because the manufacturing goods and the service goods are complementary rather than substitutable ($\epsilon < 1$). As a result, we have the faster labor growth in the service sector ($g^*_{lm} < g^*_{ls}$). The same intuition applies to the R&D expenditure. If productivity grows faster in manufacturing, R&D expenditure should balance out the slow productivity growth of the service sector in the optimal allocation. Thus, R&D expenditure in the service sector grows faster than in the manufacturing sector ($g^*_{sm} < g^*_{ss}$). Note that even in the optimal allocation, however, these counteracting labor and R&D expenditure growths fail to restore the non-balanced nature of productivity growth ($\phi_m \neq \phi_s$) to a balanced growth economy where $g^*_{am} = g^*_{as}$ and $g^*_{ym} = g^*_{ys}$.

Finally, the growth rate of the economy g^* is determined by the slow growing sector's growth rate due to the complementary nature of the economy ($\epsilon < 1$). Thus, the growth rate of the economy is increasing in ϕ_{min}. This is an intuitive result since when the idea stock effect is greater, the idea production becomes less expensive. This cheaper idea production can lead to more consumption from the less R&D expenditure or more idea production from the same R&D expenditure. Either more ideas or more consumption implies faster growth. Moreover, the growth rate of the economy is decreasing in θ, and hence is decreasing in the elasticity of substitution among ideas $1/(1-\theta)$. This decreasing relationship implies that our economy can grow faster when existing ideas are less substitutable for new ideas. It makes sense that we have faster growth when the ideas can be better integrated in general. One may think this result depreciates the role of creative destruction in economic growth. However, the creative destruction which boosts economic growth is a discrete event rather than continuous destruction of existing ideas and replacing them with the new ones. If the creative destruction is a continuous process, then it must be very costly. Thus, we can conclude that the decreasing relationship between θ and the growth rate does not conflict with the creative destruction in economic growth because the elasticity of substitution among ideas in our model captures the aggregate level of substitution in a continuous fashion.

3.3. The Competitive Equilibrium

In this section, we study the competitive equilibrium with imperfect competition in the R&D sector. We begin by stating the decision problems of the various agents in the economy, and then we put these together into the formal definition of equilibrium. We finally solve the equilibrium allocations and prices, and discuss their limiting growth rates along a CGP.

Household Problem: Given the time paths of the wage rate $\{w(t)\}$ and the interest rate $\{r(t)\}$, the representative household solves for the time path of consumption $\{C(t)\}_{t=0}^{\infty}$,

$$\max_{\{C(t)\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \frac{C(t)^{1-\sigma}-1}{1-\sigma} dt$$

$$\text{s.t.} \qquad \dot{V}(t) = r(t)V(t) + w(t)L(t) - C(t)$$

The representative household provides labor $L(t)$ and earns the wage $w(t)$. It also owns the corporate assets, the value of which is denoted by $V(t)$. We will later see that this asset value comes from the monopolistic R&D sector and $V(t) = A_m(t)v_m(t) + A_s(t)v_s(t)$ where $A_i(t)$ is productivity or the level of the idea stock in the sector i and $v_i(t)$ is the value of an idea in the sector i.

Final Goods Producer (FG): Given the time paths of $\{A_i(t), p_i(t)\}_{t=0}^{\infty}$, at each t, a competitive final goods producer solves

$$\max_{\{Y_m(t), Y_s(t)\}} Y(t) - p_m(t)Y_m(t) - p_s(t)Y_s(t)$$

$$\text{s.t.} \qquad Y(t) = \left(\gamma Y_m(t)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)Y_s(t)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{28}$$

We normalize the price of the final good to be 1. A competitive final goods producer purchases the intermediate goods $Y_m(t), Y_s(t)$ at the prices of $p_m(t), p_s(t)$ to produce the final good $Y(t)$.

Intermediate Goods Producer in the sector i (IG): Given $\{A_i(t), p_i(t), \{q_i(\nu, t)\}_{\nu \in [0, A_i(t)]}\}_{t=0}^{\infty}$, at each t, a competitive intermediate good producer in the sector i solves,

$$\max_{z_i(\nu,t)} p_i(t) \left(\int_0^{A_i(t)} z_i(\nu, t)^{\theta} d\nu\right)^{\frac{1}{\theta}} - \int_0^{A_i(t)} q_i(\nu, t) z_i(\nu, t) d\nu$$

An intermediate goods producer in the sector i purchases the machines $z_i(\nu, t)$ from the R&D sector to produce the intermediate good $Y_i(t)$. Note that productivity $A_i(t)$ is the measure of the available machine varieties as mentioned earlier. The production function takes the form of CES aggregator $Y_i(t) = \left(\int_0^{A_i(t)} z_i(\nu, t)^{\theta} d\nu\right)^{\frac{1}{\theta}}$ in (3).

R&D Sector(R&D): An R&D firm has the idea production function at each sector i,

$$\dot{A}_i(t) = \eta_i A_i(t)^{\phi_i} X_i(t) \tag{29}$$

where $X_i(t)$ denotes R&D expenditure in the unit of final goods. The monopolistic R&D firms have the perpetual exclusive ownership of an idea to produce a machine. Given the time paths of $\{w(t), r(t)\}_{t=0}^{\infty}$, an R&D firm produces machines using labor and sets the price $q_i(\nu, t)$ of a machine variety v in the sector i to maximize the present discounted value of an idea $v_i(\nu, t)$:

$$\max_{\{q_i(\nu,s)\}_{s=t}^{\infty}} v_i(\nu, t) = \int_t^{\infty} exp(-\int_t^s r(s')ds')\pi_i(\nu, s)ds$$

where $\pi_i(\nu,t) = (q_i(\nu,t) - w(t))z_i(\nu,t)$.
Note that the corresponding HJB equation is given by

$$r(t)v_i(\nu,t) = \dot{v}_i(\nu,t) + \pi(t) \tag{30}$$

In addition, we have the free entry conditions: for each sector i,

$$\eta_i A_i(t)^{\phi_i} V_i(\nu,t) \leq 1, X_i(t) \geq 0, \text{and } \eta_i A_i(t)^{\phi_i} V_i(\nu,t) = 1 \text{ if } X_i(t) > 0. \tag{31}$$

These free entry conditions determine the R&D expenditure allocation $X_m(t)$ and $X_s(t)$ across sectors.
Lastly, the market clearing conditions are as follows.
Market Clearing Conditions:

$$C(t) + X_m(t) + X_s(t) \leq Y(t), \tag{32}$$
$$\int_0^{A_i(t)} z_i(\nu,t)d\nu = L_i(t) \text{ for each } i \in \{a,m\}, \tag{33}$$
$$L_m(t) + L_s(t) = L_0 e^{nt}. \tag{34}$$

Now that these decision problems have been laid out, we are ready to define an equilibrium with imperfect competition in the R&D sector.

Definition A *competitive equilibrium with imperfect competition in the R&D sector* consists of time paths for the 1)allocations $\{Y_i(t), A_i(t), L_i(t), X_i(t), \{v_i(\nu,t), \pi_i(\nu,t), z_i(\nu,t)\}_{\nu\in[0,A_i(t)]}\}_{t=0,i\in\{m,s\}}^{\infty}$, $\{C(t), Y(t), V(t), L(t)\}_{t=0}^{\infty}$ and the 2)prices $\{r(t), w(t)\}_{t=0}^{\infty}$, $\{p_i(t), \{q_i(\nu,t)\}_{\nu\in[0,A_i(t)]}\}_{t=0,i\in\{m,s\}}^{\infty}$ such that for all t:

1. $\{C(t), V(t)\}$ solve the HH Problem.
2. $\{Y_m(t), Y_s(t)\}$ solve the FG Problem.
3. $\{z_m(\nu,t), z_s(\nu,t)\}$ solve the IG Problem.
4. $\{q_m(\nu,t), q_s(\nu,t), \pi_m(\nu,t), \pi_s(\nu,t)\}$ solve the R&D Problem.
5. $w(t)$ clears the labor market: $L_m(t) + L_s(t) = L(t)$.
6. $r(t)$ clears the asset market: $V(t) = A_m(t)v_m(t) + A_s(t)v_s(t)$.

7. $\{p_m(t), p_s(t)\}$ clears the intermediate goods market: (3).
8. $\{v_m(\nu,t), v_s(\nu,t)\}$ are determined by the HJB equations (30).
9. $\{A_m(t), A_s(t)\}$ are given by the production function in (5).
10. $\{X_m(t), X_s(t)\}$ are pinned down by the free-entry condition (31).
11. $\{L_m(t), L_s(t)\}$ satisfy the labor resource constraint (33).
12. $Y(t)$ is given by the production function in (2).
13. $L(t) = L(0)e^{nt}$
14. Using symmetry among machine variety $\nu \in [0, A_i(t)]$,
$v_i(t) \equiv v_i(\nu,t)$, $q_i(t) \equiv q_i(\nu,t)$, $z_i(t) \equiv z_i(\nu,t)$ and $\pi_i(t) \equiv \pi_i(\nu,t)$.

Notice that there are 24 equilibrium objects and 24 equations which determine these equilibrium objects at each point in time. Next, as in the previous section, we define the Constant Growth Path(CGP) as an equilibrium path where each equilibrium object grows at a constant, but possibly different rate. We again denote the growth rate $g_{ij} \equiv \frac{\dot{I}_j(t)}{I_j(t)}$, $\lim_{t\to\infty} g_{ij} = g^*_{ij}$.

First, the household maximization gives the Euler equation,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\sigma}(r(t) - \rho). \tag{35}$$

This implies that $r(t)$ should converge to a constant along a CGP, so we have

$$g^*_c = \frac{1}{\sigma}(r^* - \rho),$$

where r^* is the equilibrium interest rate in the limit along a CGP. Also, the transversality condition from the household maximization

$$\lim_{t\to\infty} e^{-\rho t} C(t)^{-\sigma} V(t) = \lim_{t\to\infty} e^{\int_0^t -r(\tau)d\tau} V(t) = 0 \tag{36}$$

should be satisfied.

Next, prices of the intermediate goods are given by (FG),

$$p_m(t) = \gamma \left(\frac{Y_m(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}}, \quad p_s(t) = (1-\gamma)\left(\frac{Y_s(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}}. \tag{37}$$

By solving (IG), we obtain the demand for machines,

$$z_i(\nu,t) = \left(\frac{q_i(\nu,t)}{p_i(t)}\right)^{\frac{1}{\theta-1}} Y_i(t). \tag{38}$$

Substituting this into (R&D) and solving the maximization problem yield

$$q_i(\nu,t) = \frac{1}{\theta} w(t). \tag{39}$$

Using the symmetry of $z_i(\nu,t)$ among machine varieties, let $z_i(t) \equiv z_i(\nu,t)$ as we did in the previous section. Then from (33),

$$z_i(t) = \frac{L_i(t)}{A_i(t)}. \tag{40}$$

This gives the derived production function for the intermediate goods

$$Y_i(t) = \left(\int_0^{A_i(t)} z_i(\nu,t)^{\theta} d\nu\right)^{\frac{1}{\theta}} = A_i(t)^{\frac{1-\theta}{\theta}} L_i(t). \tag{41}$$

Note that (9) is identical to (40), and (41) is identical to (10). From (37),(38),(39),(40) and (41),

$$w(t) = \theta p_m(t) A_m(t)^{\frac{1-\theta}{\theta}} = \theta p_s(t) A_s(t)^{\frac{1-\theta}{\theta}} \tag{42}$$

$$\rightarrow \begin{cases} w(t) = \theta\gamma \left(\frac{Y_m(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}} A_m(t)^{\frac{1-\theta}{\theta}} = \theta(1-\gamma)\left(\frac{Y_s(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}} A_s(t)^{\frac{1-\theta}{\theta}}, \\ \frac{p_m(t)}{p_s(t)} = \left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}} = \frac{\gamma}{1-\gamma}\left(\frac{Y_m(t)}{Y_s(t)}\right)^{-\frac{1}{\epsilon}}. \end{cases} \tag{43}$$

Differentiating (43) with respect to time, the relationship among the limiting growth rates of $Y_m(t)$, $Y_s(t)$, $Y_s(t)$, and $Y_s(t)$ is given by

$$g^*_{ym} - g^*_{ys} = \epsilon\frac{1-\theta}{\theta}(g^*_{am} - g^*_{as}) \tag{44}$$

Next, (43) altogether with (37) and (41) gives the labor allocation in the competitive equilibrium,

$$
\begin{aligned}
L_m(t) &= \frac{\gamma^\epsilon}{(1-\gamma)^\epsilon \left(\frac{A_s(t)}{A_m(t)}\right)^{-\frac{1-\theta}{\theta}(1-\epsilon)} + \gamma^\epsilon} L(t), \\
L_s(t) &= \frac{(1-\gamma)^\epsilon}{(1-\gamma)^\epsilon + \gamma^\epsilon \left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}(1-\epsilon)}} L(t), \\
\frac{L_m(t)}{L_s(t)} &= \frac{(1-\gamma)^\epsilon}{\gamma^\epsilon} \left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}(1-\epsilon)}. \qquad (45)
\end{aligned}
$$

Accordingly, the limiting growth rates of labor along a CGP are

$$
\begin{aligned}
&\text{if } g^*_{am} > g^*_{as}, \quad g^*_{lm} = n - \tfrac{1-\theta}{\theta}(1-\epsilon)(g^*_{am} - g^*_{as}) < g^*_{ls} = n, \\
&\text{if } g^*_{as} > g^*_{am}, \quad g^*_{lm} = n < g^*_{ls} = n - \tfrac{1-\theta}{\theta}(1-\epsilon)(g^*_{as} - g^*_{am}).
\end{aligned}
$$

First notice that this labor allocation and the limiting growth rates are identical to the optimal allocation case(see (18) and (19)). Subsequently, the asymptotic labor share convergence in (20) applies in the competitive equilibrium as well. Thus, as in the optimal allocation, the labor in the sector with slow productivity growth grows as fast as the total labor, while the labor in the sector with fast productivity growth grows slower than the total labor. In the long run, the labor share of the fast productivity growing sector reaches 0.

This identical labor allocation also implies that the growth rate of $Y(t)$ in (25) is still valid in the competitive equilibrium because $Y_i(t)$ is given by the same equation as the optimal allocation case((10), (41)).

We now specify the equilibrium R&D expenditures. First, using symmetry, the HJB equation (30) can be rewritten as

$$r(t)v_i(t) = \dot{v}_i(t) + \pi_i(t).$$

Also, plugging (39) and (41) to $\pi_i(\nu, t)$, we get

$$\pi_i(\nu, t) = \pi_i(t) = \frac{1-\theta}{\theta} \frac{L_i(t)}{A_i(t)} w(t). \qquad (46)$$

If a free entry condition holds at equality in sector i,

$$\eta_i A_i(t)^{\phi_i} v_i(t) = 1 \quad \to \quad v_i(t) = \frac{1}{\eta_i} A_i(t)^{-\phi_i}$$
$$\to \quad \dot{v}_i(t) = -\phi_i \frac{\dot{A}_i(t)}{A_i(t)} v_i(t).$$

Plugging this into (30), we obtain the value of a new idea $v_i(t)$,

$$v_i(t) = \frac{\pi_i(t)}{r(t) + \phi_i \frac{\dot{A}_i(t)}{A_i(t)}}. \tag{47}$$

This again into the free entry condition yields

$$\eta_i A_i(t)^{\phi_i} \frac{\pi_i(t)}{r(t)+\phi_i \frac{\dot{A}_i(t)}{A_i(t)}} = 1 \quad \to \quad \eta_i A_i(t)^{\phi_i} \frac{1-\theta}{\theta} \frac{L_i(t)}{A_i(t)} w(t) = r(t) + \phi_i \frac{\dot{A}_i(t)}{A_i(t)}$$
$$\to \quad \eta_i A_i(t)^{\phi_i} \frac{1-\theta}{\theta} \frac{L_i(t)}{A_i(t)} w(t) = r(t) + \phi_i \eta_i A_i(t)^{\phi_i - 1} X_i(t)$$
$$\to \quad \eta_i A_i(t)^{\phi_i - 1} \frac{1-\theta}{\theta} L_i(t) w(t) = r(t) + \phi_i g_{ai}.$$

Let us next make an assumption that the free entry condition holds in every sector. Plugging (5) and (43) into the last equation gives us the R&D expenditure in the sector i

$$X_m(t) = \frac{\gamma(1-\theta) g_{am} \left(\frac{Y_m(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}} Y_m(t)}{r(t) + \phi_m g_{am}},$$
$$X_s(t) = \frac{(1-\gamma)(1-\theta) g_{as} \left(\frac{Y_s(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}} Y_s(t)}{r(t) + \phi_s g_{as}}. \tag{48}$$

Therefore, the R&D expenditure share is,

$$\frac{X_m(t)}{X_s(t)} = \frac{\gamma^\epsilon}{(1-\gamma)^\epsilon} \frac{g_{am}}{g_{as}} \frac{r(t) + \phi_s g_{as}}{r(t) + \phi_m g_{am}} \left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}(1-\epsilon)} = \frac{g_{am}}{g_{as}} \frac{r(t) + \phi_s g_{as}}{r(t) + \phi_m g_{am}} \frac{L_m(t)}{L_s(t)}. \tag{49}$$

Again, as in the optimal allocation, the R&D expenditure ratio is inversely related to the productivity ratio. Since $\frac{A_s(t)}{A_m(t)}$ converges to 0 or ∞ on a non-balanced CGP, (49) further states that more R&D expenditure are exploited in the sector with slow productivity growth, and hence with lower productivity. That is, the lower productivity sector gives more incentive to innovate so that the sector attracts more fund than the other sector.

Differentiating (48) with respect to time, if $g^*_{am} > g^*_{as}$,

$$\begin{cases} -\frac{1}{\epsilon}(g^*_{ym} - g^*_y) + g^*_{lm} + \frac{1-\theta}{\theta}g^*_{am} = g^*_{xm} = (1-\phi_m)g^*_{am}, \\ -\frac{1}{\epsilon}(g^*_{ys} - g^*_y) + g^*_{ls} + \frac{1-\theta}{\theta}g^*_{as} = g^*_{xs} = (1-\phi_s)g^*_{as}, \end{cases}$$

$$\overset{(25)}{\longrightarrow} \begin{cases} -\frac{1}{\epsilon}(g^*_{ym} - g^*_{ys}) + g^*_{lm} + \frac{1-\theta}{\theta}g^*_{am} = (1-\phi_m)g^*_{am}, \\ g^*_{ls} + \frac{1-\theta}{\theta}g^*_{as} = (1-\phi_s)g^*_{as}, \end{cases}$$

$$\overset{(44)}{\longrightarrow} \begin{cases} g^*_{am} = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}\frac{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}n & > g^*_{as} = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}n, \\ g^*_{lm} = \left(1 - \frac{1-\theta}{\theta}\frac{\phi_m-\phi_s}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}\frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}\right)n & < g^*_{ls} = n, \\ g^*_{ym} = \left((1-\phi_s) + \epsilon\frac{\phi_m-\phi_s}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}\right)\frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}n & > g^*_{ys} = \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}}n. \end{cases} \tag{50}$$

These limiting growth rates are identical to the corresponding rates in the optimal allocation. Therefore, we again have $\phi_m > \phi_s \Leftrightarrow g^*_{am} > g^*_{as}$, and need to impose the same restriction $1 - \phi_s - \frac{1-\theta}{\theta} > 0$ to have a positive growth equilibrium.

It also follows that the limiting growth rates of the R&D expenditure $X_i(t)$ is the same as in the optimal allocation since (21) applies to the competitive equilibrium as well. From (48), the R&D expenditure shares are given by

$$\frac{X_m(t)}{X(t)} = \frac{1}{1+\frac{r(t)+\phi_m g_{am}}{r(t)+\phi_s g_{as}}\frac{1-\gamma}{\gamma}\frac{g_{as}}{g_{am}}\left(\frac{Y_m(t)}{Y_s(t)}\right)^{\frac{1-\epsilon}{\epsilon}}},$$
$$\frac{X_s(t)}{X(t)} = \frac{1}{1+\frac{r(t)+\phi_s g_{as}}{r(t)+\phi_m g_{am}}\frac{\gamma}{1-\gamma}\frac{g_{am}}{g_{as}}\left(\frac{Y_m(t)}{Y_s(t)}\right)^{-\frac{1-\epsilon}{\epsilon}}}.$$

As in (51), we know from (25) that $\frac{Y_m(t)}{Y_s(t)}$ goes to 0 if $\phi_m > \phi_s$, and to ∞ if $\phi_m < \phi_s$. Following from this result, the R&D expenditure shares converge as in (52).

Next, the same argument for the consumption growth provides the same result $g^*_c = g^*_y$, which in turn leads to the same limiting growth of the economy $g^* = g^*_c - n = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}\frac{1-\theta}{\theta}n$. To sum up, the economy under the optimal allocation and the competitive equilibrium economy grow at the exact same rate in the limit. However, one should be careful not to interpret this as the optimal allocation and competitive equilibrium having the same allocation trajectory over time. They have different transition paths and convergence speeds to the limiting values so that they do not draw the same trajectory. This can be easily seen from comparing the R&D expenditure ratio in (23) and (49).

Different from the analysis of optimal allocation in the previous subsection, the existence of market prices in the competitive equilibrium enables the discussion of consumption expenditure. If we take into account that we choose the intermediate goods aggregation into the final good as in (2) over the consumption side aggregation due to the tractability of solving the model, we can interpret $p_i(t)Y_i(t)$ as the consumption expenditure in the sector i. Let $\tilde{C}_i(t)$ denote the consumption expenditure $p_i(t)Y_i(t)$, and $\tilde{C}(t)$ denote the total consumption expenditure $\tilde{C}_m(t) + \tilde{C}_s(t)$. Then from (37), the consumption expenditure shares are given by,

$$\begin{aligned} \frac{\tilde{C}_m(t)}{\tilde{C}(t)} &= \frac{1}{1+\frac{1-\gamma}{\gamma}\left(\frac{Y_m(t)}{Y_s(t)}\right)^{\frac{1-\epsilon}{\epsilon}}}, \\ \frac{\tilde{C}_s(t)}{\tilde{C}(t)} &= \frac{1}{1+\frac{\gamma}{1-\gamma}\left(\frac{Y_m(t)}{Y_s(t)}\right)^{-\frac{1-\epsilon}{\epsilon}}}. \end{aligned} \tag{51}$$

Again, the convergence of $\frac{Y_m(t)}{Y_s(t)}$ determines the limiting behavior of consumption expenditure share, which shows the same convergence as the labor and R&D expenditure share. Therefore, the consumption expenditure share converges as follows:

$$\begin{aligned} &\text{if } \phi_m > \phi_s, \quad \lim_{t\to\infty} \frac{\tilde{C}_m(t)}{\tilde{C}(t)} = 0, \lim_{t\to\infty} \frac{\tilde{C}_s(t)}{\tilde{C}(t)} = 1, \\ &\text{if } \phi_m < \phi_s, \quad \lim_{t\to\infty} \frac{\tilde{C}_m(t)}{\tilde{C}(t)} = 1, \lim_{t\to\infty} \frac{\tilde{C}_s(t)}{\tilde{C}(t)} = 0. \end{aligned} \tag{52}$$

It is important to note that this result on the limiting growth rates in the competitive equilibrium is valid only when the free entry conditions hold. The following proposition guarantees that the free entry conditions hold asymptotically.

Proposition 1 Suppose that $\phi_m > \phi_s$, $\lim_{t\to\infty} \dot{r}(t) = 0$, and $\epsilon < 1$, then $\lim\limits_{t\to\infty} \left(v_m(t) - \frac{1}{\eta_m A_m(t)^{\phi_m}}\right) = 0$ and $\lim\limits_{t\to\infty} \left(v_s(t) - \frac{1}{\eta_s A_s(t)^{\phi_s}}\right) = 0$.

Proof:[6] The outline of the proof is as follows. For all $i \in \{m, s\}$, we first show $\limsup_{t\to\infty}\left(v_i(t) - \frac{1}{\eta_i A_i(t)^{\phi_i}}\right) = 0$, and then show $\liminf_{t\to\infty}\left(v_i(t) - \frac{1}{\eta_i A_i(t)^{\phi_i}}\right) = 0$, thereby prove $\lim_{t\to\infty}\left(v_i(t) - \frac{1}{\eta_i A_i(t)^{\phi_i}}\right) = 0$.

First notice that $\limsup_{t\to\infty}\left(v_i(t) - \frac{1}{\eta_i A_i(t)^{\phi_i}}\right) \leq 0$ and $\liminf_{t\to\infty}\left(v_i(t) - \frac{1}{\eta_i A_i(t)^{\phi_i}}\right) \leq 0$ from the free entry conditions (31). Now suppose that

$$\limsup_{t\to\infty}\left(v_m(t) - \frac{1}{\eta_m A_m(t)^{\phi_m}}\right) < 0 \text{ and } \limsup_{t\to\infty}\left(v_s(t) - \frac{1}{\eta_s A_s(t)^{\phi_s}}\right) < 0 \tag{53}$$

This implies that neither of the free entry conditions doesn't hold asymptotically, so we have

$$g^*_{am} = g^*_{as} = 0$$
$$\rightarrow g^*_{ym} = g^*_{ys} = g^*_{lm} = g^*_{ls} = g^*_y = n.$$

Then it must be also true that

$$\lim_{t\to\infty}\frac{\dot{\pi}_i(t)}{\pi_i(t)} = n.$$

From $r(t)v_i(t) = \dot{v}_i(t) + \pi_i(t)$ in (30), $\lim_{t\to\infty} v_i(t) = \infty$ since $\lim_{t\to\infty} \dot{r(t)} = 0$ along a CGP and $\lim_{t\to\infty} \pi_i(t) = \infty$, but $\dot{v}_i(t)$ cannot be $-\infty(\because v_i(t) \geq 0)$. Then, $\lim_{t\to\infty}\left(v_i(t) - \frac{1}{\eta_i A_i(t)^{\phi_i}}\right) = \infty$. This contradicts to the slack free entry assumption. Thus we cannot have (53).

Next suppose $\limsup_{t\to\infty}\left(v_s(t) - \frac{1}{\eta_s A_s(t)^{\phi_s}}\right) < 0$. Then it must be that $\limsup_{t\to\infty}\left(v_m(t) - \frac{1}{\eta_m A_m(t)^{\phi_m}}\right) = 0$ by the previous argument. This implies $g^*_{as} = 0 < g^*_{am}$, then it follows that $g^*_{ls} = n$ and $\lim_{t\to\infty}\frac{\dot{\pi}_s(t)}{\pi_s(t)} = n > 0$. Again by the same logic as before, $\lim_{t\to\infty}\left(v_s(t) - \frac{1}{\eta_s A_s(t)^{\phi_s}}\right) = \infty$, which is the contradiction. Therefore, we conclude $\limsup_{t\to\infty}\left(v_s(t) - \frac{1}{\eta_s A_s(t)^{\phi_s}}\right) = 0$. $\limsup_{t\to\infty}\left(v_m(t) - \frac{1}{\eta_m A_m(t)^{\phi_m}}\right) = 0$ can be proved analogously.

Next we prove that $\liminf_{t\to\infty}\left(v_i(t) - \frac{1}{\eta_i A_i(t)^{\phi_i}}\right) = 0$. Suppose, to derive a contradiction, that $\liminf_{t\to\infty}\left(v_m(t) - \frac{1}{\eta_m A_m(t)^{\phi_m}}\right) < 0$. This implies that there exists a recurring interval (t'_0, t'_2) such that $v_m(t) - \frac{1}{\eta_m A_m(t)^{\phi_m}} < 0$ for all $t \in (t'_0, t'_2)$. Suppose that $t \in (t'_0, t'_2)$ is unbounded. This would imply that $g^*_{am} = 0 < g^*_{as}$, yielding the same contradiction

[6]This proof is partly analogous to the proof of Lemma 3 in Acemoglu and Guerrieri (2006)

as before. Thus (t_0', t_2') must be bounded, so there exists $(t_0, t_2) \supset (t_0', t_2')$ such that for $t \in (t_0, t_2) \backslash (t_0', t_2')$, we have $v_m(t) - \frac{1}{\eta_m A_m(t)^{\phi_m}} = 0$. There will be no R&D expenditure when $t \in (t_0', t_2')$, so $A_m(t)$ remains the same. In turn, this makes $L_m(t)$ increase as well. We can also show that $\pi_m(t)$ is increasing as follows. First, $Y_s(t)$ and $Y(t)$ in terms of $Y_m(t)$ are

$$\begin{aligned} Y_s(t) &= \left(\tfrac{1-\gamma}{\gamma}\right)^{\epsilon} \left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}\epsilon} Y_m(t) \\ \rightarrow Y(t) &= \left[\gamma Y_m(t)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) Y_s(t)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \\ &= \left[\gamma + (1-\gamma)\left(\tfrac{1-\gamma}{\gamma}\right)^{\epsilon-1} \left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}(\epsilon-1)}\right]^{\frac{\epsilon}{\epsilon-1}} Y_m(t). \end{aligned}$$

Using this result, $\pi_m(t)$ is

$$\begin{aligned} \pi_m(t) &= \frac{1-\theta}{\theta}\frac{L_m(t)}{A_m(t)} w(t) \\ &= (1-\theta)\gamma \frac{L_m(t)}{A_m(t)} A_m(t)^{\frac{1-\theta}{\theta}} \left(\frac{Y_m(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}} \\ &= (1-\theta)\gamma A_m(t)^{\frac{1-\theta}{\theta}-1} L_m(t) \left[\gamma + (1-\gamma)\left(\tfrac{1-\gamma}{\gamma}\right)^{\epsilon-1}\left(\frac{A_s(t)}{A_m(t)}\right)^{\frac{1-\theta}{\theta}(\epsilon-1)}\right]^{\frac{1}{\epsilon-1}}. \end{aligned}$$

Note that the term inside the bracket [...] increases when $A_s(t)/A_m(t)$ increases. As a result, $\pi_m(t)$ increases in $t \in (t_0', t_2')$ because $L_m(t)$ and $A_s(t)/A_m(t)$ increase as discussed earlier. Next, we know from the HJB equation (30) that $\dot{v_m}(t)$ is well defined, so $v_m(t)$ is continuously differentiable. Since $A_m(t_0') = A_m(t_2')$,$1/(\eta_m A_m(t_0')^{\phi_m}) = v_m(t_0') = v_m(t_2') = 1/(\eta_m A_m(t_2')^{\phi_m}) > v_m(t)\ \forall t \in (t_0', t_2')$, $\dot{v_m}(t_0') < 0$, and $\dot{v_m}(t_2') > 0$, we have at least one $\underline{t} \in (t_0', t_2')$ such that $\dot{v_m}(\underline{t}) = 0$ so that $v_m(\underline{t})$ is a local minimum. Let t_1' to be the minimum of such $\underline{t} \in (t_0', t_2')$. Then from the HJB equation (30),

$$\begin{aligned} v_m(t_1') = \frac{\dot{v_m}(t_1') + \pi_m(t_0')}{r^*} = \frac{\pi_m(t_1')}{r^*} \quad &< \quad v_1(t_0') = \frac{\dot{v_m}(t_0') + \pi_m(t_0')}{r^*} \\ \rightarrow \underbrace{0 < \frac{\pi_m(t_1') - \pi_m(t_0')}{r^*}}_{\because\ \pi_m(t) \text{ is increasing}} \quad &< \quad \underbrace{\frac{\dot{v_m}(t_0')}{r^*} < 0}_{\because\ \dot{v_m}(t_0')<0}. \end{aligned}$$

This is a contradiction because the left-hand side is positive and the right-hand side is negative. Therefore, we must have $\liminf_{t\to\infty}\left(v_m(t)-\frac{1}{\eta_m A_m(t)^{\phi_m}}\right)=0$. We can prove $\liminf_{t\to\infty}\left(v_s(t)-\frac{1}{\eta_s A_s(t)^{\phi_s}}\right)=0$ by applying the analogous argument. This completes the proof. ■

Our final step is to check the transversality condition (36),

$$\begin{aligned}
& \lim_{t\to\infty} e^{-\int_0^t r(\tau)d\tau} V(t) \\
= & \lim_{t\to\infty} e^{-\int_0^t r(\tau)d\tau} (A_m(t)v_m(t)+A_s(t)v_s(t)) \\
= & \lim_{t\to\infty} e^{-\int_0^t r(\tau)d\tau} \left(\frac{(1-\theta)\gamma Y_m(t)\left(\frac{Y_m(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}}}{r(t)+\phi_m\frac{\dot{A}_m(t)}{A_m(t)}} + \frac{(1-\theta)(1-\gamma)Y_s(t)\left(\frac{Y_s(t)}{Y(t)}\right)^{-\frac{1}{\epsilon}}}{r(t)+\phi_s\frac{\dot{A}_s(t)}{A_s(t)}} \right) = 0.
\end{aligned}$$

This condition is satisfied when $\phi_m > \phi_s$ if

$$\begin{aligned}
& -r^* + \max\left\{g^*_{ym} - \tfrac{1}{\epsilon}(g^*_{ym}-g^*_{ys}), g^*_{ys}\right\} \\
= & -r^* + g^*_{ys} < 0 \\
\rightarrow & \ g^*_{ys} < r^* = \sigma g^*_c + \rho \qquad (\because (35)) \\
\rightarrow & \ (1-\sigma)g^*_{ys} < \rho \qquad (\because g^*_c = g^*_{ys}) \\
\rightarrow & \ (1-\sigma)\frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}} < \rho.
\end{aligned}$$

Notice that this condition is the same as (28) which we impose for the optimal allocation to meet the transversality condition of the social planner's problem.

Finally we summarize the limiting behavior of a competitive equilibrium along a CGP in the following Theorem 2.

Theorem 2. Suppose $\epsilon < 1$, $n > 0$, $(1-\sigma)\frac{1-\phi_{min}}{1-\phi_{min}-\frac{1-\theta}{\theta}}n < \rho$ and $2-\frac{1}{\theta} > \phi_{min}$. Then there exists a competitive equilibrium CGP such that

$$\text{if } \phi_m > \phi_s,$$
$$\begin{cases}
g_{am}^* = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}\frac{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}n & > g_{as}^* = \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}n \\
g_{xm}^* = \frac{1-\phi_m}{1-\phi_s-\frac{1-\theta}{\theta}}\frac{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}n & < g_{xs}^* = \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}}n \\
g_{lm}^* = \left(1-\frac{1-\theta}{\theta}\frac{\phi_m-\phi_s}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}\frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}\right)n & < g_{ls}^* = n \\
g_{ym}^* = \left((1-\phi_s)+\epsilon\frac{\phi_m-\phi_s}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}\right)\frac{1}{1-\phi_s-\frac{1-\theta}{\theta}}n & > g_{ys}^* = \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}}n \\
g_y^* = g_{ys}^* = g_c^* = g_{xs}^* = \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}}n \\
g^* = g_y^* - n = g_c^* - n = \frac{\frac{1-\theta}{\theta}}{1-\phi_s-\frac{1-\theta}{\theta}}n. \\
\lim_{t\to\infty}\frac{L_m(t)}{L(t)} = 0, \lim_{t\to\infty}\frac{L_s(t)}{L(t)} = 1 \\
\lim_{t\to\infty}\frac{X_m(t)}{X(t)} = 0, \lim_{t\to\infty}\frac{X_s(t)}{X(t)} = 1 \\
\lim_{t\to\infty}\frac{\tilde{C}_m(t)}{\tilde{C}(t)} = 0, \lim_{t\to\infty}\frac{\tilde{C}_s(t)}{\tilde{C}(t)} = 1
\end{cases}$$

if $\phi_s > \phi_m$,

$$\begin{cases} g^*_{am} = \frac{1}{1-\phi_m-\frac{1-\theta}{\theta}}n & < g^*_{as} = \frac{1}{1-\phi_m-\frac{1-\theta}{\theta}}\frac{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}n \\ g^*_{xm} = \frac{1-\phi_m}{1-\phi_m-\frac{1-\theta}{\theta}}ncv & > g^*_{xs} = \frac{1-\phi_s}{1-\phi_m-\frac{1-\theta}{\theta}}\frac{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}n \\ g^*_{lm} = n & > g^*_{ls} = \left(1-\frac{1-\theta}{\theta}\frac{\phi_s-\phi_m}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}\frac{1}{1-\phi_m-\frac{1-\theta}{\theta}}\right)n \\ g^*_{ym} = \frac{1-\phi_m}{1-\phi_m-\frac{1-\theta}{\theta}}n & < g^*_{ys} = \left((1-\phi_m)+\epsilon\frac{\phi_s-\phi_m}{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}\right)\frac{1}{1-\phi_m-\frac{1-\theta}{\theta}}n \\ g^*_y = g^*_{ym} = g^*_c = g^*_{xm} = \frac{1-\phi_m}{1-\phi_m-\frac{1-\theta}{\theta}}n & \\ g^* = g^*_y - n = g^*_c - n = \frac{\frac{1-\theta}{\theta}}{1-\phi_m-\frac{1-\theta}{\theta}}n. & \\ \lim_{t\to\infty}\frac{L_m(t)}{L(t)} = 1, \lim_{t\to\infty}\frac{L_s(t)}{L(t)} = 0 & \\ \lim_{t\to\infty}\frac{X_m(t)}{X(t)} = 1, \lim_{t\to\infty}\frac{X_s(t)}{X(t)} = 0 & \\ \lim_{t\to\infty}\frac{\tilde{C}_m(t)}{\tilde{C}(t)} = 1, \lim_{t\to\infty}\frac{\tilde{C}_s(t)}{\tilde{C}(t)} = 0 & \end{cases}$$

Proof: The derivations of the limiting growth rates and the share convergence are given in the preceding discussion in this section. ■

Theorem 1 and Theorem 2 have identical assumptions and the limiting behaviors. The only difference is that we can now discuss consumption expenditure in the competitive equilibrium. Thus, the discussion followed by Theorem 1 still applies here as well. The sector with the greater idea stock effect has faster productivity growth and hence will have the higher productivity as time goes by. Due to the complementarity, more labor demand arises in the low-productivity sector. Also, the low-productivity sector attracts more R&D expenditure to improve productivity through the channel of the incentives and complementarity, but this is not enough to lead us to a balanced economy. The price in the high-productivity sector falls fast as less labor is required. This leads to the fall in the consumption expenditure of the high-productivity sector as well despite the nominal output grows faster in this sector. The long-run growth rate of the economy is then determined by the growth rate of the low-productivity sector. The role of ϕ_{min} and θ in the growth rate of the economy is the same as in the optimal allocation.

4. Discussion

We have shown in the previous section that how the difference in the idea production function across sectors leads to non-balanced economic growth. It is worth summarizing a few important results of the optimal allocation and the competitive equilibrium of the model.

First, Theorem 2 tells us that under the assumption of $\phi_m > \phi_s$, our model shows the non-balanced pattern of economic growth where employment, consumption expenditure, and R&D expenditure grow faster in services than in manufacturing. This non-balanced growth exactly matches with the stylized facts of structural change we discussed in introduction.

Second, the optimal allocation and the competitive equilibrium have the same limiting behavior as seen in Theorem 1 and Theorem 2. Productivity of the sector with high ϕ grows faster than productivity of the other sector. Under the assumption of complementarity between the manufacturing goods and the service goods, labor and R&D expenditure are allocated in a way to fix the imbalances in productivity growth. The optimality condition in the social planner's problem and the research incentives coming from an idea ownership in the competitive equilibrium guide this balancing response of labor and R&D expenditure. Hence, labor growth and R&D expenditure growth are faster in the sector with low ϕ. However, this balancing effort is not enough to place our economy on a balanced growth path. It follows that output in the sector with low ϕ grows slower than the other sector because the labor growth in the sector is not fast enough to cancel out the slower productivity growth. As a result, the long-run growth of the economy is driven by the sector with low ϕ.

Third, the competitive equilibrium R&D expenditure is not optimal as expected from the monopolistic R&D sector, while labor in competitive equilibrium is allocated as in the optimal allocation. Suppose that both the economy under optimal allocation and the competitive equilibrium economy are on a CGP and they are now stable enough to show the limiting behavior. Let us then compare the two economies when they have the same productivity ratio $\widetilde{\frac{A_m}{A_s}}$. The two economies do not necessarily have the same level of productivity, and may not be at the same point in time when they reach at this ratio $\widetilde{\frac{A_m}{A_s}}$. We denote the time when the optimal economy and the competitive equilibrium economy reach the productivity ratio of by t' and t'' respectively. (13) and (45) imply that the labor ratio will be the same in both economies. However, the R&D expenditure will not be the same. From (23) and (49), if $\phi_m > \phi_s$, we know

$$\left(\frac{X_m(t')}{X_s(t')}\right)^{OP} > \left(\frac{X_m(t'')}{X_s(t'')}\right)^{CE} \qquad (\because \frac{r^* + \phi_s g^*_{as}}{r^* + \phi_m g^*_{am}} < 1)$$

where superscript 'OP' denotes the optimal allocation and 'CE' denotes the competitive equilibrium. The R&D expenditure ratio in the optimal allocation is greater than the ratio in the competitive equilibrium. We cannot say anything about the absolute level of R&D expenditure, but this implies that *relatively* less R&D expenditure goes to the manufacturing sector in the competitive equilibrium than it would in the optimal allocation. From this, we infer that the market gives more incentives to put more effort than optimally needed to mitigate the non-balanced nature in the idea production.

Next, our model exhibits the weak scale effects as other idea-based growth models do. That is, the long-run growth rate of the economy is an increasing function of the rate of population growth. A simple reasoning goes as follows. Labor growth leads to output growth, and growing output implies we have more to invest in R&D. This growing R&D expenditure further promotes productivity growth, and the productivity growth contributes back to the output growth. This can also be described as in the following:

$$g^* = \frac{\frac{1-\theta}{\theta}}{1 - \phi_m - \frac{1-\theta}{\theta}} n \quad \leftrightarrows \quad \frac{1-\theta}{\theta} g^*_{as} \quad \leftrightarrows \quad \frac{1}{1-\phi_s} g^*_{xs} \propto n$$

Because both inputs to production - labor and ideas(productivity) - are increasing, the increasing returns to scale production possibilities make output grows faster than labor. As a result, the output per capita and consumption per capita must grow.

Lastly, policy implications can be made on the exogenous parameters (ϕ, θ, and n) which determine the growth rate of economy. Our model implies that the long-run growth rate is invariant of policies such as a R&D subsidy. The temporal R&D subsidy to a certain sector can cause the transitional effect and hence the *level* of productivity will increase. But it cannot affect the long-run growth *rate* and the non-balanced growth pattern will remain. This suggests that only the policies which affect ϕ, θ, and n can increase the long-run growth. We can increase the population growth n using usual fertility promotion policies. The effect of a policy on θ and hence the elasticity of substitution among ideas $1/(1-\theta)$ can be very ambiguous and hard to measure. Furthermore, making ideas less substitutable may increase the long-run growth, but many times we benefit from the revolutionizing new ideas.

Then, what about ϕ? ϕ is the measure of the positive impact of the existing idea stock on the new idea production. We can also think of ϕ as the measure of knowledge spillover or synergy effect within a sector. When the technologies in one sector are strongly related to each other, new idea production will be more likely to depend on the existing stock of ideas. Thus, directing research to learn from the existing technologies in other industries may increase ϕ. The policies targeting this aim should increase the *permanent* tendency to learn from other industries.

One possible example of the permanent increase in ϕ can be found in the post-1995 period. In Figure 3, we see the R&D expenditure growth slowdown in the service sector after 1995. Along with this, a couple studies(Desmet and Rossi-Hansberg (2010) and Triplett and Bosworth (2004)) document the post-1995 labor productivity growth surge in the service sector. One might argue that this trend contradicts the our model predictions, but we can still explain the trend with our model. It can be a transitional behavior, or what possibly happened in the post-1995 period is that Internet or Information technology(IT) completely changed the fundamental idea production in the service sector. That is, the advancement in IT increased the knowledge spillover effect within service industries, and hence this led to the bigger ϕ_s. This in turn implies the faster productivity growth and less R&D expenditure growth in services, which we see in the post-1995 period.

This idea of increasing the long-run growth rate by the permanent change in ϕ is consistent with Cozzi (1997) who shows that it is possible to change the long-run growth rate by shifting research towards the greater spillover effect. But we should be careful that the policies to increase ϕ may decrease the R&D expenditure and may affect other variables in the model as well. It is important to note that changing exogenous parameters can affect other endogenous variables at the same time.

5. Simple Calibration

In this section, we conduct a simple calibration exercise to see if the calibrated model generates the growth rates consistent with empirical results from the literature on the labor productivity growth. We use NIPA data(1929-2009) for GDP and the sectoral employment and consumption expenditure. We also use NSF data for the R&D expenditure. See Appendix A for the detailed data description.

We first adopt the following parameters from data:

- 2% annual discount rate, $\rho = 0.02$
- The growth rate of Full-Time Equivalent employment in manufacturing and service, $n = 0.0184$
- 3.33% annual asymptotic total output growth rate from 1929 to 2009, $g_y^* = 0.0333$

We also take $g_{xm}^* = 0.0192$ obtained from the NSF data. The growth rate of R&D expenditure in manufacturing shows the fairly stable growth as seen in Figure 3. In contrast, the R&D expenditure in service sector doesn't seem to show the stability until the late 1990s. But from the late 1990s, the growth rate of the R&D expenditure exhibits the growth rate of 0.0317 which is a very close number to $g_y^* = 0.0333$. Since the asymptotic growth rate is our main interest, we will just set $g_{xs}^* = g_y^* = 0.0333$. Next, we also have $g_{lm}^* = 0.091$ from NIPA data, and $g_{ls}^* = 0.0181$ is very close to $n = 0.0184$ as our model predicts so. From these numbers, it is clear that we have the $\phi_m > \phi_s$ case.

Now we calibrate ϕ_m, ϕ_s, and θ by varying $\epsilon < 1$ to have the minimum sum of differences in the following equations.

$$\begin{aligned}
g_{xm}^* &: \left| 0.0192 - \frac{1-\phi_m}{1-\phi_s-\frac{1-\theta}{\theta}} \frac{1-\phi_s+\frac{1-\theta}{\theta}(1-\epsilon)}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)} n \right| \\
g_{xs}^* = g_y^* &: \left| 0.0333 - \frac{1-\phi_s}{1-\phi_s-\frac{1-\theta}{\theta}} n \right| \\
g_{lm}^* &: \left| 0.091 - \left(1 - \frac{1-\theta}{\theta} \frac{\phi_m-\phi_s}{1-\phi_m+\frac{1-\theta}{\theta}(1-\epsilon)} \frac{1}{1-\phi_s-\frac{1-\theta}{\theta}} \right) n \right|
\end{aligned}$$

Note that from (10) we can infer the growth rate of idea stock g_{am}^* and g_{as}^* are related

Table 1: Labor Productivity Calibration

Labor Prod. Growth (annual %)	Model		Chang and Hong	Wölfl	
	$\epsilon = 0.5$, (ϕ_m, ϕ_s, θ) $= (0.66, 0.35, 0.77)$	$\epsilon = 0.8$, (ϕ_m, ϕ_s, θ) $= (0.63, 0.36, 0.78)$	1958-1996	1980-1990	1990-2000
Manufacturing	2.45	2.45	2.71	3.56	4.15
Service	1.50	1.50	N/A	0.61	1.57

to the labor productivity. In particular, $\frac{1-\theta}{\theta} g^*_{am}$ and $\frac{1-\theta}{\theta} g^*_{as}$ in the model serve as the labor productivity growth. There are a few sets of local minimizers (ϕ_m, ϕ_s, θ), but they all give us the same labor productivity growth in the model. Model calibration with a couple benchmark ϵ values are shown in Table 1. as well as the previous results from Chang and Hong (2006) and Wölfl (2003). Chang and Hong (2006) studies the labor productivity growth of the manufacturing sector, and Wölfl (2003) mainly focuses on the productivity growth in service industries. Overall, our model estimates fall into a reasonable range which do not conflict with these two empirical studies. The magnitudes in our model estimates are somewhat smaller. This may be due to the fact that the sector definition in our study and the other two studies are not exactly the same. Especially, our manufacturing sector includes the mining and construction industries whereas Wölfl (2003) separately reports these industries' labor productivity growth separately. In Wölfl (2003), these industries show slower labor productivity growth than the other manufacturing industries. Thus, the smaller estimates in our model are in fact expected.

Also, one may wonder where we are now on the equilibrium path. For example, we may ask how far we have gone in the manufacturing labor share convergence to 0. This requires the dynamic simulation of the model, which we leave for future research.

6. Conclusion

Employment, consumption expenditure and R&D investment in the service sector have grown faster than those in the manufacturing sector. In an effort to understand this sectoral shift, this paper shows that difference in the idea production across sectors can be a driving force behind the non-balanced sectoral growth. We construct an endogenous two-sector growth model where the current stock of ideas is an input to the new idea production but the idea stock effect (ϕ in the model) varies across sectors. Under the complementarity assumption between sectors, the greater idea stock effect in manufacturing induces R&D investment to move towards the service sector as a balancing reaction against the different productivity growth. But this reallocation of R&D investment is not big enough for the service sector to catch up with the productivity growth in the manufacturing sector. This in turn leads to the labor and consumption expenditure shifts to the service sector, hence the economy features the non-balanced growth.

This paper adds to the supply side explanations of the non-balanced growth literature, and the unique contribution is that it further explores the fundamental source of different productivity growth by having endogenous technological progress. This endogenous technology also enables us to generate the R&D expenditure shift to the service sector while previous approaches account for the same trend only in employment and consumption expenditure.

The non-balanced growth is clearly not the result of a single factor and the model studied here leaves out other factors such as non-homothetic preference which may be important to the non-balanced growth. This suggests a future research direction toward a unified approach. Lastly, it would be useful to empirically prove that the idea production in manufacturing is indeed more dependent on the current stock of ideas than the idea production in services.

A Appendix: Data Sources

The sources of data used in Figure 1, Figure 2, and Section 5 are described in this appendix.

A1. Employment and Consumption Expenditure Data

We use NIPA tables from the BEA for the U.S. employment and consumption expenditure by sector during 1929-2009. For the employment, we use data from NIPA Table 6.5: "Full-Time Equivalent Employees by Industry". And for the consumption expenditure, NIPA Table 2.4.5: "Personal Consumption Expenditures by Type of Product" and Table 3.10.5: "Government Consumption Expenditures and General Government Gross Output" are used. These two tables are in current dollars, thus we converted the numbers to constant 2005 dollars using GDP and population data in Table 7.1: "Selected Per Capita Product and Income Series in Current and Chained Dollars".

Using this data, we then follow the sectoral assignment in Herrendorf, Rogerson and Valentinyi (2009):

- Agriculture: "food and beverages purchased for off-premises consumption"
- Manufacturing: "durable goods"; "nondurable goods" excluding "food and beverages purchased for off-premises consumption"
- Services: "services"; "government consumption expenditure"

Note that we drop the agricultural sector in Section 5.

A2. R&D Expenditure Data

Private R&D expenditure data is from National Science Foundation(NSF). Annual R&D expenditure during 1969-1998 is available from Table H-23 in the Survey of Industry Research and Development(SIRD). We construct the sectoral R&D expenditure series from 1984 because R&D expenditure by industry is not available in earlier years. NSF replaced SIRD with the Business Research and Development and Innovation Survey(BRDIS) in 2008, and BRDIS includes annual R&D expenditure data from 1999. But we dropped the data from 1999 to 2003 due to the inconsistency in the industry classification. The industry classification changed from the SIC system to NAICS in 1999, and NAICS assigned quite a few companies to the industry which is not consistent with their primary

R&D activity. NSF started to manually revise this industry assignment from 2004. Both SIRD and BRDIS provide the first-order classification of industries as manufacturing or non-manufacturing, in which the non-manufacturing sector is composed of service industries. Thus we follow this classification for the sectoral assignment.

B Appendix: The Two Sector Model with Scientists

In this section, we lay out a two-sector endogenous growth model with scientists creating new ideas in contrast to the lab-equipment model in the main text. We will consider the broad set of parameter values of the elasticity of substitution between the manufacturing goods and the service goods including the Cobb-Douglas utility case. We omit details of the proof here.

B1. The Economic Environment

Each sector i has final good producers, intermediate good producers, and scientists. There also exist standard households. First, the basic production functions in this economy are these:

$$Y_i(t) = \left(\int_0^{A_i(t)} z_i(\nu, t)^{\theta} d\nu \right)^{\frac{1}{\theta}}, \tag{54}$$

$$\int_0^{A_i(t)} z_i(\nu, t) d\nu = L_i(t), \tag{55}$$

$$\dot{A}_i(t) = \eta_i A_i(t)^{\phi_i} S_i(t) \tag{56}$$

(54) and (55) are the same as in the two sector model discussed in the main sections. The idea production function (56) is now different. New ideas are produced by scientists $S_i(t)$ who work over the existing stock of knowledge rather than the R&D expenditure. ϕ_i captures the effect of the existing stock of knowledge on the new idea production - both standing on shoulders effect and the fishing out effect. We again assume $0 < \phi_i < 1$.

Next, resource constraints for this economy are:

$$L_i(t) = \int_0^{A_i(t)} x_i(\nu,t)d\nu,$$
$$Y_i(t) = c_i(t)L(t),$$
$$L_a(t) + L_m(t) \equiv L(t) = L_0 e^{n_l t},$$
$$S_a(t) + S_m(t) \equiv S(t) = S_0 e^{n_s t}$$

where $c_i(t)$ is the real consumption (not the total or nominal expenditure) per capita, n_l is the growth rate of total labor and n_s is the growth rate of the number of total scientists. Note that since there is no capital in the economy, the representative household consumes all the output produced.

The representative household has the following preference:

$$U_t = \int_t^{\infty} e^{-\rho\tau} u(c_m(\tau), c_s(\tau))d\tau$$

$$u(C_m(\tau), C_s(\tau)) = \frac{((\gamma C_m(\tau)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma)C_s(\tau)^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}})^{1-\sigma}}{1-\sigma} \tag{57}$$

where ϵ is the elasticity of substitution between the manufacturing goods and the service goods and $1/\sigma$ is the elasticity of intertemporal substitution. Note that the elasticity of intertemporal substitution doesn't play a role since we don't have the capital in the model. When $\epsilon \to 1$ this utility function becomes the Cobb-Douglas, so the utility function is then $u(C_m(\tau), C_s(\tau)) = \frac{(C_m(\tau)^{\gamma} C_s(\tau)^{1-\gamma})^{1-\sigma}}{1-\sigma}$.

B2. The Optimal Allocation

Given this economic environment, let's first consider the optimal allocation by solving the social planner's problem.

Social Planner's Problem(SP): the social planner sets the optimal paths of

$$\{C_i(t), Y_i(t), L_i(t), S_i(t), \{x_i(\nu,t)\}_{v=0}^{A_i(t)}\}_{t=0, i\in\{m,s\}}^{\infty}$$

which solves

$$\max_{\{C_m(t), C_s(t)\}} U_t = \int_t^{\infty} e^{-(\rho)\tau} u(C_m(\tau), C_s(\tau))d\tau$$

s.t.

$$Y_i(t) = \left(\int_0^{A_i(t)} x_i(\nu,t)^\theta d\nu \right)^{\frac{1}{\theta}} = C_i(t), \forall i \in \{m,s\},$$
$$\int_0^{A_i(t)} x_i(\nu,t) d\nu = L_i(t), \forall i \in \{m,s\},$$
$$\dot{A}_i(t) = \eta_i A_i(t)_i^\phi S_i(t), \forall i \in \{m,s\},$$
$$L_m(t) + L_s(t) = L(t)$$
$$S_m(t) + S_s(t) = S(t)$$

Using the symmetry among intermediate goods, we have $x_i(\nu,t) = x_i(t)$. Thus, we get $L_i(t) = \int_0^{A_i(t)} x_i(\nu,t)d\nu = A_i(t)x_i(t)$, and $C_i(t) = Y_i(t) = \left(\int_0^{A_i(t)} x_i(t)^\theta d\nu\right)^{\frac{1}{\theta}} = \left(A_i(t)x_i(t)^\theta\right)^{\frac{1}{\theta}} = A_i(t)^{\frac{1}{\theta}-1}L_i(t)$.
Then SP can be rewritten as in the following.

Social Planner's Problem(SP'): the social planner sets the optimal path of

$$\{C_i(t), L_i(t), S_i(t)\}_{t=0, i\in\{m,s\}}^{\infty}$$

which solves

$$\max_{\{C_m(t), C_s(t)\}} U_t = \int_t^\infty e^{-(\rho)\tau} u(C_m(\tau), C_s(\tau)) d\tau$$

s.t.

$$C_i(t) = A_i(t)^{\frac{1-\theta}{\theta}} L_i(t), \forall i \in \{m,s\},$$
$$\dot{A}_i(t) = \eta_i A_i(t)^{\phi_i} S_i(t), \forall i \in \{m,s\},$$
$$L_m(t) + L_s(t) = L(t)$$
$$S_m(t) + S_s(t) = S(t)$$

We first set up the current-value Hamiltonian,

$$\begin{aligned}\mathcal{H} = \\ & u(A_m(t)^{\frac{1-\theta}{\theta}} L_m(t), A_s(t)^{\frac{1-\theta}{\theta}} (L(t) - L_m(t))) \\ & + \mu_m(t)\eta_m A_m(t)^{\phi_m} S_m(t) \\ & + \mu_s(t)\eta_s A_s(t)^{\phi_s} (S_0 e^{n_s t} - S_m(t))\end{aligned}$$

And we have the transversality conditions

$$\lim_{t\to\infty} e^{-\rho t}\mu_m(t)A_m(t) = 0 \tag{58}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu_s(t)A_s(t) = 0 \tag{59}$$

Let us first consider the Cobb-Douglas utility case. Along the any optimal path,

$$\frac{A_m(t)^{\frac{1-\theta}{\theta}}}{A_s(t)^{\frac{1-\theta}{\theta}}} = \frac{1-\gamma}{\gamma}\frac{C_m(t)}{C_s(t)}$$

Plugging in $C_m(t) = A_m(t)^{\frac{1-\theta}{\theta}}L_m(t)$ and $C_s(t) = A_s(t)^{\frac{1-\theta}{\theta}}L_s(t)$ in the equation above, we obtain the optimal allocations of labor as,

$$\frac{L_m(t)}{L(t)} = \gamma + \frac{1-\gamma}{A_m(t)^{\frac{1-\theta}{\theta}}},$$

$$\frac{L_s(t)}{L(t)} = 1-\gamma - \frac{1-\gamma}{A_m(t)^{\frac{1-\theta}{\theta}}}$$

Thus, the labor share of manufacturing decreases over time as its technology advances and converges to γ. Accordingly, the labor share of service increases over time and converges to $1-\gamma$.

Along a CGP, we obtain the limiting allocation of researchers between the two sectors as the following:

$$\left(\frac{S_m}{S_s}\right)^* = \kappa\frac{\eta_s}{\eta_m}\frac{1-\phi_s}{1-\phi_m} \text{ for some constant } \kappa \tag{60}$$

Also, the limiting growth rate of each variable is given in the following:

$$g^*_{cm} = n_l + \frac{1-\theta}{\theta}\frac{n_s}{1-\phi_m}, g^*_{cs} = n_l + \frac{1-\theta}{\theta}\frac{n_s}{1-\phi_s},$$

$$g^*_{lm} = g^*_{ls} = n_l, g^*_{sm} = g^*_{ss} = n_s,$$

$$g^*_{am} = \frac{n_s}{1-\phi_m}, g^*_{as} = \frac{n_s}{1-\phi_s}$$

As a result, we have a balanced growth in the labor allocation and the scientists allocation across sectors when we have Cobb-Douglas utility although productivity growth is unbalanced.

Next, consider the general functional form of utility as in equation (57). Along the any optimal path,

$$\frac{A_m(t)^{\frac{1-\theta}{\theta}}}{A_s(t)^{\frac{1-\theta}{\theta}}} = \frac{1-\gamma}{\gamma}\left(\frac{C_m(t)}{C_s(t)}\right)^{1/\epsilon}$$

Plugging in $C_m(t) = A_m(t)^{\frac{1-\theta}{\theta}} L_m(t)$ and $C_s(t) = A_s(t)^{\frac{1-\theta}{\theta}} L_s(t)$ in the equation above, we obtain the optimal allocations of labor at any point in time along the optimal path:

$$\frac{L_s(t)}{L(t)} = \frac{(1-\gamma)^\epsilon}{\gamma^\epsilon (A_s(t)/A_m(t))^{\frac{1-\theta}{\theta}} + (1-\gamma)^\epsilon}$$

Then, the optimal labor share at any point in time along the optimal path is,

$$\frac{L_s(t)}{L(t)} = \frac{(1-\gamma)^\epsilon}{\gamma^\epsilon (A_s(t)/A_m(t))^{\frac{1-\theta}{\theta}(1-\epsilon)} + (1-\gamma)^\epsilon}$$

Thus, the labor share of service sector converges to $\dfrac{(1-\gamma)^\epsilon}{\delta\gamma^\epsilon + (1-\gamma)^\epsilon}$ if $\lim_{t\to\infty}(A_s/A_m)^{\frac{1-\theta}{\theta}(1-\epsilon)}$ is some constant δ, to 1 if the limit goes to 0, and to 0 if the limit goes to infinity.

This limit depends on ϕ_m, ϕ_s, θ, and ϵ. Also, these parameters determine which limiting growth pattern along the optimal CGP we would follow. There are three cases of limiting growth patterns along the optimal CGP depending on these parameters: one balanced growth path and two non-balanced growth paths. This is summarized below:

CASE 1: A Balanced Growth

If $\phi_m = \phi_s = \phi$, the limiting asymptotic growth rates are:

$$\begin{aligned}
g^*_{am} &= g^*_{as} = \frac{n_s}{1-\phi}, \\
g^*_{cm} &= g^*_{cs} = n_l + \frac{1}{1-\phi}\frac{1-\theta}{\theta} n_s, \\
g^*_{lm} &= g^*_{ls} = n_l, \\
g^*_{sm} &= g^*_{ss} = n_s
\end{aligned}$$

Now, let $\beta_m \equiv (1-\epsilon)\frac{1}{1-\phi_m}\frac{1-\theta}{\theta}$ and $\beta_s \equiv (1-\epsilon)\frac{1}{1-\phi_s}\frac{1-\theta}{\theta}$.

CASE 2:

If (1) $\epsilon < 1$ and $\phi_m < \phi_s$, or

(2) $\epsilon > 1, -1 < \beta_m < \beta_s (\Longleftrightarrow \beta_m > -1, \beta_s > -1, \phi_m > \phi_s)$, or

(3) $\epsilon >> 1, -1 > \beta_m > \beta_s (\Longleftrightarrow \beta_m < -1, \beta_s < -1, \phi_m < \phi_s)$

the limiting asymptotic growth rates are:

$$g^*_{am} = \frac{n_s}{1-\phi_m}, \qquad g^*_{as} = \frac{1}{1-\phi_s}\frac{1+\beta_m}{1+\beta_s}n_s,$$
$$g^*_{cm} = n_l + \frac{1-\theta}{\theta}\frac{1}{1-\phi_m}n_s, \qquad g^*_{cs} = n_l + \frac{1-\theta}{\theta}\frac{1}{1-\phi_m}n_s\left(\frac{(1-\phi_m)(1+\beta_m)}{(1-\phi_s)(1+\beta_s)}\epsilon + (1-\epsilon)\right)$$
$$g^*_{lm} = n_l, \qquad g^*_{ls} = n_l + \frac{\beta_m-\beta_s}{1+\beta_s}n_s,$$
$$g^*_{sm} = n_s, \qquad g^*_{ss} = \frac{1+\beta_m}{1+\beta_s}n_s.$$

CASE 3:

If (1) $\epsilon < 1$ and $\phi_m > \phi_s$, or

(2) $\epsilon > 1, \beta_m > \beta_s > -1 (\Longleftrightarrow \beta_m > -1, \beta_s > -1, \phi_m < \phi_s)$, or

(3) $\epsilon >> 1, \beta_m < \beta_s < -1 (\Longleftrightarrow \beta_m < -1, \beta_s < -1, \phi_m > \phi_s)$

the limiting asymptotic growth rates are:

$$g^*_{am} = \frac{1}{1-\phi_m}\frac{1+\beta_s}{1+\beta_m}n_s, \qquad g^*_{as} = \frac{n_s}{1-\phi_s},$$
$$g^*_{cm} = n_l + \frac{1-\theta}{\theta}\frac{1}{1-\phi_s}n_s\left(\frac{(1-\phi_s)(1+\beta_s)}{(1-\phi_m)(1+\beta_m)}\epsilon + (1-\epsilon)\right), \qquad g^*_{cs} = n_l + \frac{1-\theta}{\theta}\frac{1}{1-\phi_s}n_s,$$
$$g^*_{lm} = n_l + \frac{\beta_s-\beta_m}{1+\beta_m}n_s, \qquad g^*_{ls} = n_l,$$
$$g^*_{sm} = \frac{1+\beta_s}{1+\beta_m}n_s, \qquad g^*_{ss} = n_s.$$

These two non-balanced growth patterns are again divided each into two cases which lead to different direction of the non-balanced growth path. This is summarized in the table below:

CASE 2	(1)$\epsilon < 1$, $\phi_m < \phi_s$	(2)$\epsilon > 1$, $-1 < \beta_m < \beta_s(\phi_m > \phi_s)$, (3)$\epsilon >> 1$, $-1 > \beta_m > \beta_s(\phi_m < \phi_s)$
	$g^*_{am} < g^*_{as}$	$g^*_{am} > g^*_{as}$
	$g^*_{cm} < g^*_{cs}$	$g^*_{cm} > g^*_{cs}$
	$g^*_{lm} > g^*_{ls}$	$g^*_{lm} > g^*_{ls}$
	$g^*_{sm} > g^*_{ss}$	$g^*_{sm} > g^*_{ss}$
CASE 3	(1)$\epsilon < 1$, $\phi_m > \phi_s$	(2) $\epsilon > 1$, $\beta_m > \beta_s > -1(\phi_m < \phi_s)$, (3)$\epsilon >> 1$, $\beta_m < \beta_s < -1(\phi_m > \phi_s)$
	$g^*_{am} > g^*_{as}$	$g^*_{am} < g^*_{as}$
	$g^*_{cm} > g^*_{cs}$	$g^*_{cm} < g^*_{cs}$
	$g^*_{lm} < g^*_{ls}$	$g^*_{lm} < g^*_{ls}$
	$g^*_{sm} < g^*_{ss}$	$g^*_{sm} < g^*_{ss}$

In CASE 2.(1) and CASE 3.(1) where elasticity of substitution is less than 1, in the limit, consumption and productivity grows faster in the sector with larger ϕ. And more labor and scientists work in the slow growing sector.

In CASE 2.(2) and CASE 3.(2) where elasticity of substitution is greater than 1, but not as big as to make $\beta_i = (1-\epsilon)\frac{1}{1-\phi_m}\frac{1-\theta}{\theta}$ less than -1, in the limit, consumption and productivity grows faster in the sector with larger ϕ. And more labor and scientists work in the fast growing sector.

In CASE 2.(3) and CASE 3.(3) where elasticity of substitution is big enough to make β_i less than -1, in the limit, consumption and productivity grows faster in the sector with smaller ϕ. And more labor and scientists work in the fast growing sector.

CASE 3 is of greater interest since the labor share and the research employment share of the service sector grow faster as discussed in the first section. Especially, to see the intuition for CASE 3.(1), suppose that we have a greater *the standing on shoulders effect* in the manufacturing sector than in the service sector ($\phi_m > \phi_s$). That is, new ideas in the manufacturing are built more dependent on the existing ideas, while new ideas in the service sector can be created in a *relatively* independent way in the macro level. Then it's more likely to have faster idea accumulation in the manufacturing sector than the service sector ($g^*_{am} > g^*_{as}$) since we can easily build upon existing knowledge in the manufacturing sector. This higher productivity growth in the manufacturing in turn leads to the faster production growth in the manufacturing sector($g^*_{cm} > g^*_{cs}$, note that c here is *not* the total *expenditure*).

Next, because the manufacturing goods and the service goods are complementary rather than substitutable($\epsilon < 1$), the different productivity growths cause the share of labor in the fast growing sector to fall to balance out the two sectors. As a result, we have the faster labor growth in the service sector ($g^*_{lm} < g^*_{ls}$). The same intuition applies to the number of scientists. If the growth rate of productivity in the manufacturing sector is faster by nature, the number of scientists reacts in a way to balance out the productivity growth of the service sector in the optimal allocation. Thus, the number of scientists in the service sector grows faster than in the manufacturing sector($g^*_{sm} < g^*_{ss}$).

References

Acemoglu, D. and V. Guerrieri, "Capital Deepening and Non-balanced Economic Growth," *NBER Working Paper*, 2006. w12475.

_ and _ , "Capital Deepening and Non-balanced Economic Growth," *Journal of Political Economy*, 2008, *116*, 467–498.

Baumol, W.J, "Macroeconomics of unbalanced growth: the anatomy of urban crisis," *American Economic Review*, June 1967, *57* (3), 415–26.

Buera, Francisco Javier and Joseph P. Kaboski, "Scale and the Origins of Structural Change," *Working paper*, 2008. University of California at Los Angeles and Ohio State University.

Chang, Y. and J. H. Hong, "Do Technological Improvements in the Manufacturing Sector Raise or Lower Employment?," *American Economic Review*, 2006, *96* (1), 352–68.

Cozzi, G., “Exploring Growth Trajectories,” *Journal of Economic Growth*, 1997, *2* (4), 385–399.

Desmet, K. and E. Rossi-Hansberg, “Spatial Development,” *NBER Working Paper*, 2010. w15349.

Engel, E., “Die Productions-und Consumptionsverhältnisse des Königreichs Sachsen,” *Zeitschrift des Statistischen Büreaus des Königlich Sächsischen Ministeriums des Inneren*, November 1857, (8 and 9). It was reprinted in *Bulletin de l'Institut International de Statistique*, IX (1895).

Foellmi, R. and J. Zweimuller, “Structural change, Engels consumption cycles, and Kaldors facts of economic growth,” *Journal of Monetary Economics*, 2008, *55*, 1317–1328.

Herrendorf, B., R. Rogerson, and A. Valentinyi, “Two Perspective on Preferences and Structural Transformation,” *NBER Working Paper*, 2009. w15416.

Jones, Charles I., “Growth and Ideas,” in P. Aghion and S.N. Durlauf, eds., *Handbook of Economic Growth*, Amsterdam: Elsevier, 2005, chapter 10, pp. 1063–1111.

Kongsamut, P., R. Sergio, and D. Xie, “Beyond Balanced Growth,” *The Review of Economic Studies*, Oct 2001, *68* (4), 869–882.

Kuznets, Simon, “Economic Growth and Income Inequality,” *The American Economic Review*, 1955, *45* (1), 1–28.

_ , “Modern Economic Growth: Findings and Reflections,” *The American Economic Review*, 1973, *63* (3), 247–258.

Matsuyama, K., “The Rise of Mass Consumption Societies,” *Journal of Political Economy*, 2002, *110*, 1035–1070.

Ngai, L. R. and C. A. Pissarides, “Structural Change in a Multisector Model of Growth,” *American Economic Review*, March 2007, *97.*

_ and R. M. Samaniego, “Accounting for Research and Productivity Growth Across Industries,” *CEP Discussion Paper*, 2009. No. 914.

Thurow, Lester C., "The Post-Industrial Era Is Over," *New York Times*, September 1989.

Triplett, J. E. and B. P. Bosworth, *Productivity in the U.S. Services Sector: New Sources of Economic Growth*, Brookings Institution Press, Washington, D.C., 2004.

Wölfl, A., "Productivity Growth in Service Industries: An Assessment of Recent Patterns and the Role of Measurement," *OECD Science, Technology and Industry Working Papers*, 7 2003.